Traffic Grooming in Path, Star, and Tree Networks: Complexity, Bounds, and Algorithms

Rudra Dutta
dutta@csc.ncsu.edu

Shu Huang
shuang5@unity.ncsu.edu

George N. Rouskas
rouskas@csc.ncsu.edu

Department of Computer Science, North Carolina State University, Raleigh, NC

1. INTRODUCTION

In wavelength division multiplexing (WDM) optical networks, the *lightpath* is the main transport element. The set of lightpaths defines a *logical topology.* The logical topology design problem has been studied extensively in the literature [2]. Traffic grooming is a variant of the logical topology design problem, and is concerned with the development of techniques for combining low speed traffic components onto high speed channels in order to minimize network cost, and has received attention in recent literature [4, 3, 1, 6, 8]. In this paper we consider the problem of traffic grooming in path, star, and tree networks, which are useful topologies in themselves but are also building blocks for more general topologies. First, we settle the complexity of traffic grooming in path and star networks. Since routing and wavelength assignment in these two topologies is trivial, these results demonstrate that traffic grooming is itself an inherently difficult problem. Secondly, we obtain a series of lower and upper bounds which are increasingly tighter but have considerably higher computational requirements, with corresponding heuristics. Out objective is to minimize the *total amount of electronic switching at all networking nodes.* A formulation of traffic grooming as an integer linear problem can be found in [4].

2. COMPLEXITY RESULTS

Below we state our theoretical results regarding traffic grooming in elemental topologies. For brevity, we omit the proofs and detailed discussions, most of which, as well as other corollaries, can be found in [7]. We consider a network in the form of a unidirectional path $\mathcal{P}$ with N nodes. There is a single directed fiber link from node i to node $i+1$, for each $i \in \{1, 2, \cdots, N-1\}$. For bidirectional paths, each directed fiber link is replaced with a directed fiber pair. We also consider a network in the form of a star $\mathcal{S}$ with $N+1$ nodes. There is a single *hub* node which is allowed to routed traffic, and is connected to every other node by a physical link. Each physical link consists of a fiber in each direction, and each fiber can carry W wavelengths. A tree network is defined as the obvious generalization of star networks, and a ring network is defined as in [3]. We note that the wavelength assignment problem trivially disappears for path and star networks.

An instance of the traffic grooming problem is provided by specifying a number of path (or star) nodes, a traffic matrix $T = [t_{ij}], 1 \leq i < j \leq N$, a grooming factor C, a number of wavelengths W, and a goal F. Each wavelength channel can carry C units of traffic. The (decision) problem asks whether a valid logical topology may be formed and all traffic in T routed over the lightpaths of the logical topology so that the total electronic switching over all nodes is less than or equal to F.

Bifurcated routing of traffic may or may not be allowed. Specifically, if bifurcation is not allowed, then for any source-destination pair (i, j) such that $t_{ij} \leq C$, we require that all t_{ij} traffic units be carried on the *same* sequence of lightpaths from source i to destination j. If bifurcation is allowed, a traffic component t_{ij} may be split into various subcomponents which may follow different logical routes from source to destination. The bifurcation is restricted to integer subcomponents.

THEOREM 2.1. *The decision version of the traffic grooming problem in both unidirectional and bidirectional path networks (both the cases of bifurcated routing of traffic allowed and not allowed) is NP-Complete.*

COROLLARY 2.1. *The decision version of the traffic grooming problem in both unidirectional and bidirectional ring networks (both the cases of bifurcated routing of traffic allowed and not allowed) is NP-Complete, even when every node has full wavelength conversion capability.*

THEOREM 2.2. *The decision version of traffic grooming in star networks is NP-Complete.*

COROLLARY 2.2. *The decision version of traffic grooming in tree networks is NP-Complete, even when every interior tree node has full wavelength conversion capability.*

3. BOUNDS AND ALGORITHMS

Path Bounds: We examine the unidirectional path with bifurcation allowed, which is the practically interesting case. In [3], a sequence of upper and lower bounds on the optimal solution for ring networks was obtained by decomposing the ring into segments, and we apply the same approach. The decomposition is effected by considering certain path nodes to be completely *opaque*, i.e., insisting that no lightpaths optically pass through these nodes. For details, the reader is referred to [3]. Successive bounds, increasingly tighter, are obtained by increasing the size of the segments allowed between opaque nodes. The upper bounds represent solutions to the traffic grooming problem whose performance is

SIGMETRICS'03, June 10–14, 2003, San Diego, California, USA.
ACM 1-58113-664-1/03/0006.

precisely characterized. In particular, when only two-hop segments are allowed, the logical topology is such that either all the odd-numbered or the even-numbered nodes are opaque, i.e., it consists of either single-hop or two-hop lightpaths. It is straightforward to verify (refer also to [3]) that the upper bound in this case is no worse than one-half the worst-case amount of the electronic switching, when no optical routing is performed.

Path Heuristic: First we note that when a traffic component is of magnitude C or more, it will have a direct lightpath carrying C units of it in an optimal solution, thus it is safe to assign such a lightpath in our heuristic approach, and reduce the traffic component accordingly. Consider the completely *transparent* topology obtained by assigning a lightpath to *each* non-zero traffic component t_{ij} of the reduced traffic matrix. If this logical topology is feasible, i.e., does not violate the wavelength limit, then it is also optimal. Accordingly, we propose the following heuristic approach: if the transparent topology for the current traffic matrix is not feasible, pick one traffic component and reassign it to a sequence of others with which it can be routed or "clubbed", then check the resultant transparent topology again.

With proper choices of (a) the traffic component to pick and (b) its rerouting, the algorithm is guaranteed to terminate with a feasible solution in a polynomial number of steps, and performs very well. If the rerouting is performed by preferentially switching traffic electonically at nodes which would be opaque for the two-hop upper bound described above, then this heuristic is also guaranteed to perform at least as well as that bound; this and other assertions above are verified numerically [7].

Star Bounds and Heuristics: Since the hub node is the only node that routes traffic either optically or electronically, any solution to a star network instance can be specified as a masking of the traffic matrix indicating whether each traffic component is routed optically or not. Optically routing a traffic component will dedicate a wavelength on the two links it traverses to it, so this decision affects the decisions for other traffic components. In particular, after dedicating the wavelengths required for the traffic components which must be routed optically for any source node, the remaining wavelengths must be sufficient to carry all other traffic sourced by that node to the hub node. A similar condition applies for destination nodes.

We obtain upper and lower bounds on the optimal that are easily computed by introducing the concept of a *partial* solution in which some of the mask elements are left unspecified, then defining an *optimistic* and a *pessimistic* completion of a partial solution. We pick the traffic components in some order. The choice of the mask element to assign to them (optical or electronic) can be viewed as a tree branching. An exhaustive search amounts to generating the whole tree, and a search terminated at progressively deeper levels provides a series of upper and lower bounds when completed pessimistically and optimistically, respectively. By specifying the order of picking the traffic components, an increasing tightness of the successive upper bounds can be guranteed. This is useful since the upper bounds are feasible solutions and can also serve as heuristic solutions. We also define a greedy heuristic which assigns lighpaths to traffic components in order of decreasing magnitude, whenever the constraints above allow. The performance of the greedy heuristic is seen to be quite good.

Tree Bounds: Our approach to the tree network is again based on decomposition, this time into star networks. Each interior node of the tree can consider the traffic that passes through it in isolation and this results in star networks. Optimal or heuristic solutions to the star networks thus formed can be reconciled into a single feasible tree solution as long as at least one of every two adjacent interior tree nodes is made opaque. We note that picking the optimal set of such interior nodes (to apply star solutions for) is itself the NPC problem of finding a maximal independent set in a graph [5]. Fortunately, easier choices can still give good results. In particular, utilizing the alternate levels of a level order traversal of the tree nodes to pick the star hub nodes guarantees an independent set, and also guarantees that the electronic switching for the resulting solution is no more than 50% of the completely opaque solution. Instead of using the optimal star solutions, the increasingly better (and feasible) upper bounds of the star networks can be used to obtain a sequence of bounds for the tree. A similar approach yields lower bounds for the tree network as well. Once again, the upper bounds are heuristic solutions.

Tree Heuristics: We also define two greedy heuristics on the tree network. Both are based on picking the traffic components in decreasing order of magnitude, and then attempting to assign them to direct lightpaths. A greedy "first-available" heuristic is used for wavelength assignment. The behavior of the two heuristic approaches differs with respect to traffic components which cannot be assigned a direct lightpath either due to lack of dedicated bandwidth or due to a wavelength clash. In the more simplistic version, the traffic component is then electronically routed at each intermediate node, but the more sophisticated one attempts to route it optically at some if not all intermediate nodes. Numerical investigation shows that both greedy heuristic algorithms perform well.

4. REFERENCES

[1] A. L. Chiu and E. Modiano. Traffic grooming algorithms for reducing electronic multiplexing costs in WDM ring networks. *Journal of Lightwave Technology*, 18(1):2–12, Jan. 2000.

[2] R. Dutta and G. N. Rouskas. A survey of virtual topology design algorithms for wavelength routed optical networks. *Optical Networks*, 1(1):73–89, January 2000.

[3] R. Dutta and G. N. Rouskas. On optimal traffic grooming in WDM rings. *Proc. ACM SIGMETRICS*, pp. 164–174, June 2001.

[4] R. Dutta and G. N. Rouskas. Traffic grooming in WDM networks: Past and future. *IEEE Network*, 16(6):46–56, 2002.

[5] M. R. Garey and D. S. Johnson. *Computers and Intractability*. W. H. Freeman and Co., New York.

[6] O. Gerstel, R. Ramaswami, and G. Sasaki. Cost-effective traffic grooming in WDM rings. *IEEE/ACM Trans. Network.*, 8(5):618–630, 2000.

[7] S. Huang. *Traffic Grooming in Wavelength Routed Path Networks*. North Carolina State University. *M.S. thesis*, 2002. *http://www.lib.ncsu.edu/ETD-db/*

[8] K. Zhu and B. Mukherjee. Traffic grooming in an optical WDM mesh network. *IEEE JSAC*, 20(1):122–133, Jan 2002.