Design of Large-Scale Optical Networks *

Yufeng Xin, George N. Rouskas, Harry G. Perros
Department of Computer Science, North Carolina State University, Raleigh NC 27695
E-mail: {yxin,rouskas,hp}@eos.ncsu.edu

ABSTRACT

We consider the problem of designing a network of optical cross-connects (OXCs) to provide end-to-end lightpath services to large numbers of label switched routers (LSRs). We present a set of heuristic algorithms to address the combined problem of physical topology design and logical topology design. Unlike previous studies which were limited to small topologies, we have applied our algorithms to networks with hundreds or thousands of LSRs and with a number of lightpaths that is an order of magnitude larger than the number of LSRs. We present numerical results for up to 1000 LSRs and for a wide range of system parameters such as the number of wavelengths per fiber, the number of transceivers per LSR, and the number of ports per OXC.

KEY WORDS

Optical Networks, Topology Design, Genetic Algorithms

1 Introduction

The wide deployment of point-to-point wavelength division multiplexing (WDM) transmission systems in the Internet infrastructure has enhanced the need for faster switching at the core of the network. At the same time, there is a growing effort to modify the Internet Protocol to support different levels of Quality of Service (QoS). Label Switching Routers (LSRs) supporting Multi-Protocol Label Switching (MPLS) [1] are being deployed to address these two issues. On one hand, LSRs simplify the forwarding function, making it possible to operate at higher data rates. On the other hand, MPLS enables the Internet architecture to behave in a connection-oriented fashion that is more conducive to supporting QoS.

The rapid evolution of optical technologies makes it possible to move beyond point-to-point WDM systems to optical backbone networks that eliminate the need for per-hop packet forwarding. Such a network consists of a number of optical cross-connects (OXCs), arranged in some arbitrary topology, and provides interconnection to a number of LSRs. Each OXC can switch the optical signal coming in on a wavelength of an input fiber link to the same wavelength in an output fiber link. The main mechanism of transport in such a network is the lightpath, which is a communication channel established between two OXCs or two LSRs and which may span a number of fiber links (physical hops). The Internet Engineering Task Force (IETF) is investigating the use of Generalized MPLS (GMPLS) [3] to set up and tear down lightpaths. GMPLS is an extension of MPLS that supports multiple types of switching, including switching based on wavelengths usually referred to as Multi-Protocol Lambda Switching (MPλS), and we will use the term "MPλS network" to refer to an optical network of OXCs. With GMPLS, the OXC backbone and the network of LSRs will share common functionality in the control plane, making it possible to seamlessly integrate OXC backbones within the overall Internet infrastructure.

The problem of designing wavelength-routed networks of OXCs has received considerable attention in the last decade [2]. Most of the work in the literature focuses on the configuration design and the routing and wavelength assignment (RWA) problems, under the assumption of a fixed fiber physical topology. That is, given a network traffic demand and a physical network topology, an optimal virtual topology and an optimal routing and wavelength assignment are obtained. In the case in which the set of lightpaths is also given, the problem is reduced to a pure RWA problem, which can be further decomposed into a routing sub-problem and a wavelength assignment sub-problem.

In this paper we consider the problem of designing large-scale optical WDM networks of OXCs that provide end-to-end lightpath services. The scale of the optical backbone network is characterized by the number of LSRs using its services and the number of lightpaths that it can support. We are interested in typical national or international networks, in which the number of LSRs can be in the hundreds, and the number of lightpaths can be an order of magnitude greater than the number of LSRs. Given the number of LSRs, the number of wavelengths per fiber link, and a set of physical constraints (such as the number of transceivers at each LSR and the number of input/output ports at each OXC), we address both the physical topology design problem (i.e., the number of OXCs required and their interconnectivity) and the routing and wavelength assignment problem. Since the problem is NP-hard, we present a set of heuristic algorithms to obtain a near-optimal solution in terms of the number of required OXCs, including a genetic algorithm to search the space of physical topologies.

Our work differs from previous studies in several important ways. To the best of our knowledge, this is the first time that the problem of designing the physical *and* logical topology of a wavelength-routed network is fully formulated and solved. Also, whereas previously published

*This work was supported by the NSF under grant ANI-9805016.

algorithms have been applied to small networks (e.g., 10- to 20-node topologies such as the NSFNet) with few (less than one hundred) lightpaths, we consider large-scale networks of realistic size. Finally, we provide new insight into the design of WDM backbone networks by investigating the effect of various system parameters and using realistic ranges for their values.

In Section 2 we describe the problem we study as well as the assumptions we make, and in Section 3 we present heuristic algorithms for generating a 2-connected graph, routing lightpaths, and assigning wavelengths. We also describe the genetic algorithm used for searching the space of physical topologies. We present numerical results in Section 4, and we conclude the paper in Section 5.

2 Problem Definition

We consider a number N of LSRs that are to be interconnected over an optical backbone network which consists of OXC nodes supporting GMPLS. The service provided by the MPλS network of OXCs is the establishment of lightpaths among pairs of LSRs. We assume that each LSR has Δ optical transceivers, therefore, it may establish at most Δ incoming and at most Δ outgoing lightpaths at any given time. This constraint on the number of simultaneous lightpaths to/from an LSR is due both to optical hardware and cost limitations (reflected in the number of optical transceivers) and the traffic processing capacity of the LSR. We also assume that each fiber link in the network can support at most W wavelengths, and that each OXC has exactly P input/output ports. We let α be the degree of connectivity of the physical topology of OXCs:

$$\alpha = \frac{E}{M(M-1)/2} \tag{1}$$

where E represents the number of fiber links interconnecting the OXCs. Parameter α represents how dense the graph is. For an arbitrary graph, α ranges from 0 to 1, with 0 representing a graph that is totally disconnected and 1 representing a completely connected graph. We note that most existing backbone networks have an α value around 0.3.

The fundamental question we address in this paper is: *What is the minimum number M of OXCs required to support the N LSRs, and what is the physical topology of the corresponding MPλS network?* We believe that the answer to this question is of importance to service providers who need to deploy optical backbone networks in a cost-effective manner. We note that the cost of building an MPλS network will be mainly determined by: **(i)** the cost of the OXCs (including switch hardware and switch controller software), and **(ii)** the cost of (deploying or leasing) the fiber links between OXCs (including the cost of related equipment, such as optical amplifiers). While in our study we directly model only the OXC cost, we note that the fiber cost is indirectly taken into account through the parameter α. Since α is an input parameter in our formulation, we believe that α in combination with the number of OXCs is representative of the overall cost of the MPλS network.

Clearly, in order to determine the number of OXCs in the backbone we need to take into account not only the number N of LSRs but also the traffic requirements (i.e., the number of lightpaths between pairs of LSRs), the survivability properties of the network, etc. Rather than trying to account for all possible design requirements, we are instead interested in providing a general framework that can help us answer the above question in a way that can provide practical guidelines for building MPλS networks. We therefore set the following requirements **R1**-**R5** that the MPλS network we design must satisfy.

R1. Each LSR accesses the backbone using two bidirectional fiber links, one to each of two different OXCs.

R2. The physical topology of the OXCs is 2-connected.

R3. Each LSR has 2Δ lightpaths to/from other LSRs.

R4. Two neighbor OXCs in the physical topology are interconnected by one bidirectional fiber link.

R5. The OXCs do not have wavelength converters.

The first two requirements (**R1** and **R2**) ensure that there are at least two edge-disjoint paths between any two LSRs, a necessary condition for a survivable network. **R3**-**R5** can be viewed as worst case requirements. **R3** ensures that the physical topology of the MPλS network can support the maximum number of simultaneous lightpaths (recall that no LSR can have more than Δ outgoing and Δ incoming lightpaths). In particular, we use $\mathcal{L}$ to denote the set of ΔN lightpaths that the network must support. Because of **R4**, the resulting network will use single-fiber links between pairs of adjacent OXCs. Finally, **R5** requires that a lightpath be assigned a single wavelength along all the physical links it traverses. **R4** and **R5** can be easily relaxed, but are included here because we believe that there will exist MPλS networks which will satisfy one or both of these requirements, at least during early deployment. Furthermore, we expect that relaxing either **R4** or **R5** will lead to a topology with a smaller number of OXCs compared to when both are in place, therefore, our results can be used as a worst-case scenario.

3 The Heuristic Algorithm

Our objective is to determine the optimal physical topology of OXCs for establishing the given set $\mathcal{L}$ of lightpaths among the N LSRs, under the constraint on the number of wavelength W that can be supported in each fiber. This involves determining the minimum number of OXCs required as well as the links interconnecting the OXCs. Because of the difficulty of this problem, we choose an indirect approach to obtaining a near-optimal physical topology. Specifically, we first assume that the number M of

OXCs in the physical topology is given and there is no constraint on the number of wavelengths. Our objective then is to obtain:

1. the set of fiber links interconnecting the M OXC nodes (i.e., the physical topology), and
2. the routing and wavelength assignment for the lightpaths in the set $\mathcal{L}$ among the N LSRs

such that the required number of wavelengths per fiber link in the physical topology is minimized. The inputs to the problem are: the number of OXCs, M; the number of ports in each OXC, P; and the static traffic matrix $[v_{s,d}]$, where $v_{s,d}$ represents the number of lightpaths that have to be established between OXCs s and d, $s, d = 1, 2, \cdots, M$. This traffic matrix is derived directly from the lightpath set $\mathcal{L}$ as follows: if a lightpath needs to be set up from LSR A connected to OXC s and LSR B connected to OXC d, then we increment $v_{s,d}$.

The solution to the above problem gives an optimal topology for M OXC nodes, as well as the optimal lightpath routing and wavelength assignment that minimizes the number W_f of wavelengths used in any link. Let $W_f^\star$ be this optimal number of wavelengths. We note that $W_f^\star$ can be greater than, equal to, or less than the number of wavelengths W actually supported by the fiber links of the MPλS network. If $W_f^\star = W$, then the solution is not only optimal, but it is also feasible given the available number of wavelengths W. However, if $W_f^\star > W$, then this optimal solution is not feasible. In this case, we may have to increase the number M of OXCs that was given as input to the problem. By solving the same problem with a larger value of M, we will obtain a new optimal solution requiring a smaller number of wavelengths. On the other hand, if $W_f^\star < W$, the solution is feasible, but it may also be possible that another solution exist, one in which the physical topology consists of a smaller number of OXCs and which requires no more than W wavelengths. Thus, we can solve the same problem with a smaller value for M as input in the hope of finding such a solution.

The above observations naturally lead to a binary search approach to obtaining an optimal solution that requires no more than W wavelengths *and* minimizes the number of OXCs in the MPλS network. The binary search is illustrated in the overall algorithm shown in Figure 1.

An integer programming model of the problem presented can only be solved for very small size networks. In this section, we present a set of heuristic algorithms for this problem that can be applied to medium and large size networks. We divide the problem into the following tasks: (i) generation of a feasible physical topology, (ii) routing of lightpaths, and (iii) assignment of wavelengths to lightpaths. Each of these tasks is solved using a heuristic algorithm. We then use a genetic algorithm (GA) to generate additional feasible physical topologies, and we iterate in order to obtain a near-optimal solution with a minimum total number of wavelengths.

3.1 Random Feasible Physical Topologies

Recall that a feasible OXC network is at least 2-connected. In order to get a feasible topology, we first generate a random tree, then grow a 2-connected graph from it. If we number the leaves of the tree as $i, \cdots, M$, we can sequentially connect pairs of leaves with edges $(i, i+1), \cdots, (M-1, M)$ to obtain a 2-connected graph. In order to generate a random tree of M nodes, we have adopted the method from [4]. Each tree of M nodes has an one-to-one relationship with a Prufer number that has $(M-2)$ digits. The digits are integers between 1 and M. Therefore, the following steps summarize the algorithm for generating a random feasible physical topology:

1. Given the number M of OXCs, randomly generate $(M-2)$ digits in the range of 1 to M to form $P(\mathcal{T})$.
2. Generate a tree $\mathcal{T}$ using the Prufer number $P(\mathcal{T})$.
3. Construct a 2-connected graph by adding edges sequentially to connect the leaves of the tree $\mathcal{T}$.

3.2 Routing and Wavelength Assignment

We now assume that we are given a 2-connected physical topology of M OXCs, as well as a set of lightpaths between pairs of OXCs that need to be established. We now present two algorithms, one to route each lightpath over a physical path of fiber links, and another to assign wavelengths to the lightpaths. Note that we treat the routing and wavelength assignment subproblems independently; this approach may require a larger number of wavelengths than a combined solution, but the latter is intractable while our approach can be applied directly to networks of realistic size.

We use Dijkstra's shortest path algorithm to route the set of lightpaths over the given physical topology. In order to minimize the number of wavelengths used on a physical link, we use two heuristic approaches. First, the link weight used in Dijkstra's algorithm is dynamically adjusted to reflect the number of wavelengths already allocated on each link. Consider physical link ℓ and let C_ℓ be the actual link cost (C_ℓ is a constant) and w_ℓ be the number of lightpaths already using this link. Then, each time we run Dijkstra's algorithm to find a path for a certain lightpath, we use the quantity $L_\ell = C_\ell + H w_\ell$ as the cost of link ℓ, where H is a tunable weight parameter. This cost function forces new lightpaths to be routed over less congested links in the physical topology, reducing the total number of wavelengths used in the network. For the next application of the algorithm, quantities w_ℓ are incremented for all links ℓ along the path of the just routed lightpath.

The second heuristic approach has to do with the order in which we consider the given lightpaths for routing. Specifically, we first sort the OXC nodes in an ascending order according to their degree (ties are broken arbitrarily). Starting with the first OXC node (the one with the smallest degree), we apply Dijkstra's algorithm to route all lightpaths that have this node as source or destination. We proceed in this manner by considering nodes with higher degrees. This method results to a considerably lower wavelength usage than when selecting the nodes randomly. This is because nodes with smaller degrees have fewer alternative links to route their lightpaths. In view of this, routing lightpaths originating or terminating at these nodes first will increase the wavelength use of their links. Because of the cost function described above, later lightpaths will tend to avoid the links around these nodes. On the other hand, if a node with a small degree was considered late in the process, its lightpaths would have to use one of its links regardless of how congested these links were, potentially increasing the overall number of wavelength required. We refer to this scheme as the smallest-degree-first-routing (SDFR) algorithm. The following steps summarize the heuristic algorithm for routing lightpaths:

1. Use the link weight function $L_\ell = C_\ell + Hw_\ell$.

2. Sort the nodes in an ascending order of their degree.

3. Consider each node in this order and use the Dijkstra's algorithm to build the shortest path for its lightpaths.

Once the physical links for each lightpath have been obtained, we need to assign wavelengths such that if two lightpaths share the same link then they are assigned a different wavelength. This wavelength assignment problem can be shown to be equivalent to the vertex coloring problem of an induced simple graph [5]. A heuristic algorithm was developed in [5] to solve the vertex coloring problem. The algorithm uses a greedy approach to assign wavelengths to the lightpaths. We adopt this algorithm to perform wavelength assigned, since it has been shown to have good accuracy and to run in polynomial time.

3.3 The Genetic Algorithm (GA)

Genetic algorithms (GAs) [4] are a practical and robust optimization and search tool. These algorithms are based on the mechanisms of evolution and natural genetics that lead to the survival of the fittest by the process of natural search and selection. A GA generates a sequence of populations using a selection mechanism, and then applies crossover and mutation as search mechanisms. A GA is a global search technique in the solution space of the problem and it usually avoids entrapping into a local optimization. The steps involved in a GA are as follows:

1. Design an efficient bit string encoding scheme.

2. Generate an initial generation of feasible solutions of size G_s.

3. If the stop criterion has been met, return the best solution in the current generation as the near-optimal solution and stop; otherwise, continue to Step 4.

4. Randomly select solutions from the current generation as the basis of the *offspring* generation.

5. Perform crossover on the selected solutions. This crossover is controlled by the *crossover rate* R_c.

6. Perform mutation on the solutions obtained from Step 5. Mutation is controlled by the *mutation rate* R_m.

7. Calculate the fitness value for the new individuals.

8. Repeat Steps 4-6 to obtain a new generation. Set this as the current generation. Go back to Step 3.

We use a GA to generate feasible physical topologies starting from the initial physical topology we obtained in Section 3.1. The objective is to search for physical topologies that will improve on the number of wavelengths required to establish the given set of lightpaths. The fitness value of a feasible individual is the number of wavelengths W_f required in the corresponding solution. We now proceed to describe the encoding of the solution, the calculation of the fitness value, the selection, crossover, and mutation strategies, and the handling of infeasible solutions.

Encoding. Consider a graph with M vertices. We number the vertices from 0 to $M-1$ and the edges of the graph from 0 to $\frac{M(M-1)}{2}-1$. We read the string from left to right. Each edge (i,j) is numbered using an index k, which is defined according to the index of the two endpoints of the edge, i and j. Specifically, $k = \frac{M(M-1)}{2} - \frac{(M-i)(M-i-1)}{2} + j - i - 1,\ 0 \le i < j < M$. The solution is encoded into a chromosome (i.e., a bit string of length $M(M-1)/2$) as follows. Each edge is represented by a bit in the bit string. If the edge exists, the bit is set to 1; otherwise it is set to 0.

Fitness value. This is the value of the objective function of the current solution (physical topology), i.e., the total number of wavelengths used in the network.

GA parameters. The performance of a GA is determined by the right choice of control parameters: the crossover rate R_c, the mutation rate R_m, and the population size, G_s. In order to shorten the running time of the algorithm, we choose a relatively small population size and therefore a high level of string disruption. We set $R_c = 0.8$, $R_m = 0.1$, and $G_s = 25$.

Infeasible individuals. It is possible that an offspring generated by the above process correspond to an infeasible graph. That is, the graph may not be 2-connected or it may exceed the constraint on the connectivity α. Therefore, we run a number of tests for each generated offspring to ensure that it represents a feasible solution. If the solution passes the tests, we run the RWA heuristics to obtain the minimum number of wavelengths needed in the network.

The complete algorithm is shown in Figure 1. A detailed complexity analysis is omitted, but Steps 6-18 take time $O(M^4)$ for an iteration of the binary search in which the number of OXCs is M.

Complete Algorithm for Designing an MPλS Network
Input: The number N of LSRs, the set $\mathcal{L}$ of lightpaths, the number P of ports per OXC, the number of wavelengths W in each fiber, the OXC connectivity α, and the number of generations G for the genetic algorithm.
Output: A physical topology of M OXCs, and the routing and wavelength assignment for the set $\mathcal{L}$ of lightpaths.

1. begin
2. $M_{max} \leftarrow N/2$, $M_{min} \leftarrow 1$
3. $num\_gen \leftarrow 0$
4. $M \leftarrow (M_{max} + M_{min})/2$ // Binary search
5. $phy\_top \leftarrow$ initial physical topology generated using the algorithm described in Section 3.1
6. For the physical topology phy_top:
7. Solve the RWA problem (see Section 3.2)
8. Calculate the objective function (fitness value) W_f
9. While $num\_gen < G$: // Genetic algorithm
10. $num\_gen \leftarrow num\_gen + 1$
11. Use the genetic algorithm in Section 3.3 to create a new generation of physical topologies
12. For each individual phy_top in the new generation calculate the fitness value W_f using Steps 7 and 8
13. $W^\star \leftarrow$ smallest fitness value W_f from last generation
14. $phy\_top^\star \leftarrow$ physical topology corresponding to $W^\star$
15. If $W^\star > W$ then
16. $M_{min} \leftarrow M$; go back to Step 3
17. Else if $W^\star < W - 5$ then
18. $M_{max} \leftarrow M$; go back to Step 3
19. Else return M, $phy\_top^\star$
20. end of the algorithm

Figure 1. Algorithm for the design of MPλS networks

4 Numerical Results

In this section we present results that illustrate how the different design parameters affect the number of OXCs required to interconnect a set of LSRs. We set the upper bound on the degree of connectivity α of the MPλS network of OXCs to 0.4. The set $\mathcal{L}$ of lightpaths is chosen so that each LSR has exactly Δ incoming and Δ outgoing lightpaths to a random set of other LSRs.

In Figure 2 we plot the number M of OXCs in the MPλS network against the number N of LSRs. Three plots are given for $W = 32, 64, 128$. For these results, we have let $\Delta = 12$ and $P = 64$. The number M of OXCs increases almost linearly with the number N of LSRs, but the slope of the curves is moderate. In particular, an increase by a factor of ten in the number of LSRs results in an increase in the number of OXCs by a factor between four and seven. Since the degree of connectivity is around 0.4, the corresponding increase in the number of links, as N increases, is similar. This result implies that OXC networks to interconnect very large number of LSRs can be built cost-effectively. Also, the larger the number of wavelengths, the smaller the number of OXCs required for a given number N of LSRs, as expected. However, a two-fold increase in the number of wavelengths (from 32 to 64) reduces the number of OXCs by less than 1/2, while a second two-fold increase (from 64 to 128) results in a smaller reduction in the number of OXCs. Note that, for a given value of N, the number of lightpaths in the set $\mathcal{L}$ remains constant at ΔN across the three curves in Figure 2, so one would expect a larger decrease in the number of OXCs as the number of wavelengths increases. However, recall that $2N$ OXC ports are needed to attach the LSRs to the OXC network. Since the number of ports per OXC remains constant at $P = 64$, as W increases, the number of OXCs needed is constrained by the number of ports required. Thus, in order to take full advantage of the larger number of wavelengths in the fiber, OXCs with a larger number of ports must be employed.

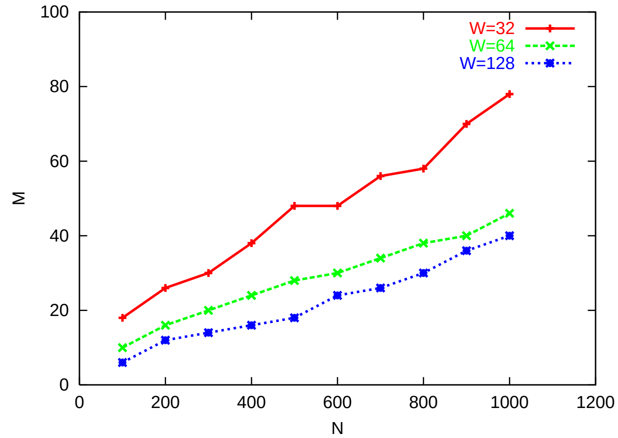

Figure 2. Number M of OXCs ($\Delta = 12$, $P = 64$)

In Figure 3 we let $W = 64$ and $P = 64$, and we plot the number of OXCs against the number N of LSRs. Three curves are shown, one for a different value of the number of transceivers per LSR, $\Delta = 4, 8, 12$. Note that the curve for $\Delta = 12$ is identical to the middle curve of Figure 2 for $W = 64$, although the scale in the two figures is different. Again, we see that the number of OXCs increases linearly with the number of LSRs. Recall that the number ΔN of lightpaths increases linearly with Δ. However, the curves show that the number of OXCs for a given N value needed to support the larger number of lightpaths increases much more slowly than Δ. These results indicate that a small incremental cost (in terms of OXCS and fiber links) can provide a significantly richer connectivity among the LSRs.

In Figure 4 we plot the number of OXCs against the number of wavelengths for $N = 300$ and $P = 64$. Three curves for different numbers of transceivers per LSR are shown, $\Delta = 4, 8, 12$. The results are as expected. Specifically, the number of OXCs needed decreases as the number W of wavelengths per fiber increases, but the curves flatten

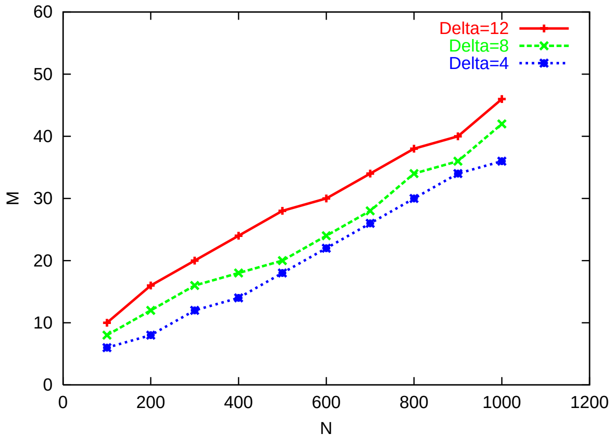

Figure 3. Number M of OXCs ($W = 64$, $P = 64$)

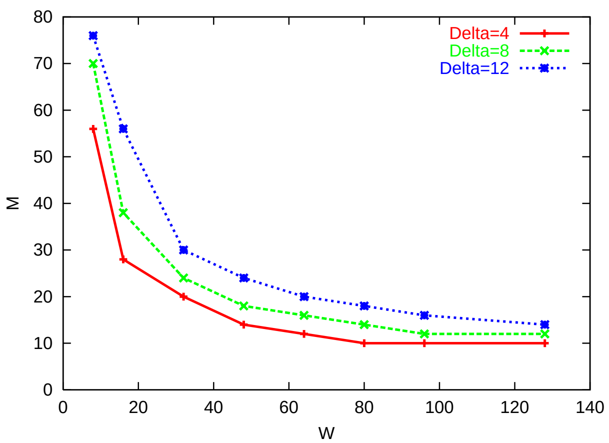

Figure 4. Number M of OXCs ($N = 300$, $P = 64$)

out once $W > 80$. As we mentioned above, this reflects the fact that a larger number of ports per OXC is needed to take full advantage of the large number of wavelengths. Also, more OXCs are required as Δ increases, but the results are consistent with the previous figure in that the increase in the number of OXCs is significantly slower than the increase in Δ (and the corresponding increase in the number of lightpaths to be established).

Finally, in Figure 5 we compare the results of our heuristic algorithm to a lower bound on the number of OXCs, which can be found in [6]. The figure plots the middle curve of Figure 2 and the corresponding lower bound; thus, these plots correspond to the following values of the input parameters: $\Delta = 12$, $P = 64$, and $W = 64$. As we can see, the two curves have very similar behavior, and the results from our heuristic are close to the lower bound. We emphasize that the lower bound is obtained simply by counting the number of network resources that are absolutely necessary to support the given set of lightpaths. In other words, there is no guarantee that the lower bound corresponds to a feasible solution.

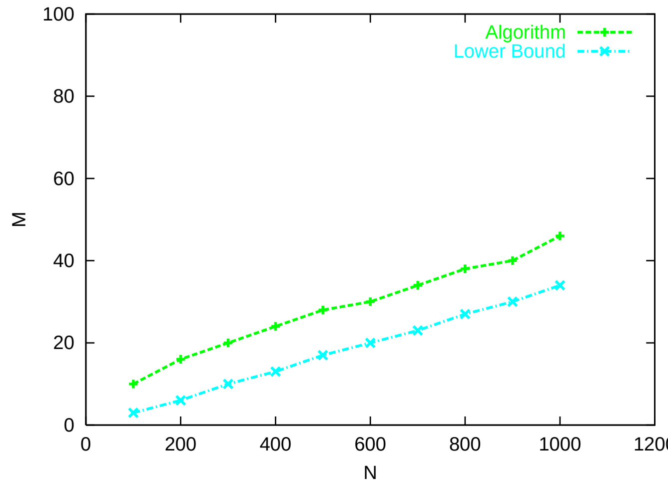

Figure 5. Heuristic vs. lower bound ($\Delta = 12$, $P = 64$, $W = 64$)

5 Concluding Remarks

We have described a set of heuristic algorithms for the physical and logical topology design of large-scale optical networks of OXCs. We presented routing and wavelength assignment heuristics, as well as a genetic algorithm to iterate over the set of physical topologies. We have applied our algorithms to design networks that can accommodate hundreds of LSRs and several thousands of lightpaths. The most important finding is that it is possible to build cost-effective networks that provide rich connectivity among the LSRs with relatively few, but properly dimensioned, OXCs.

References

[1] B. Davie and Y. Rekhter. *MPLS Technology and Applications*. Morgan Kaufmann Publishers, 2000.

[2] R. Dutta and G. N. Rouskas. A survey of virtual topology design algorithms for wavelength routed optical networks. *Optical Networks*, 1(1):73–89, Jan. 2000.

[3] P. Ashwood-Smith *et al.* Generalized MPLS – signaling functional description. IETF Draft, April 2001.

[4] C.C. Palmer and A. Kershenbaum. An approach to a problem in network design using genetic algorithms. *Networks*, 26:151–163, 1995.

[5] Z. Zhang and A. Acampora. A heuristic wavelength assignment algorithm for multihop WDM networks with wavelength routing and wavelength reuse. *IEEE/ACM Transactions on Networking*, 3(3):281–288, June 1995.

[6] Yufeng Xin. *Topology Design of Large-Scale Optical Networks*. PhD thesis, NCSU, Raleigh, NC, 2002.