OSN: 8 DTD 5 pp. 1–18 (col. fig: NIL)

Available online at www.sciencedirect.com

SCIENCE DIRECT®

ELSEVIER

Optical Switching and Networking xx (xxxx) xxx–xxx

Optical Switching and Networking

www.elsevier.com/locate/osn

Traffic grooming in WDM ring networks to minimize the maximum electronic port cost

Bensong Chen*, George N. Rouskas, Rudra Dutta

Department of Computer Science, North Carolina State University, Raleigh, NC 27695-7534, United States

Received 20 July 2004; received in revised form 7 December 2004; accepted 31 January 2005

Abstract

We consider the problem of traffic grooming in WDM ring networks. Traffic grooming is a variant of the well-known logical topology design problem, and is concerned with the development of techniques for combining low speed traffic components onto high speed channels in order to minimize network cost. Previous studies have focused on *aggregate* representations of the network cost. In this work, we consider a *Min–Max objective*, in which it is desirable to minimize the cost at the node where this cost is maximum. Such an objective is of high practical value when dimensioning a network for unknown future traffic demands and/or for dynamic traffic scenarios. We present new theoretical results which demonstrate that traffic grooming with the Min–Max objective is NP-complete even when wavelength assignment is not an issue. We also present new polynomial-time traffic grooming algorithms for minimizing the maximum electronic port cost in both unidirectional and bidirectional rings. We evaluate our algorithms through experiments with a wide range of problem instances, by varying the network size, number of wavelengths, traffic load, and traffic pattern. Our results indicate that our algorithms produce solutions which are always close to the optimal and/or the lower bound, and which scale well to large network sizes, large number of wavelengths, and high loads. We also demonstrate that, despite the focus on minimizing the maximum cost, our algorithms also perform well in terms of the aggregate electronic port cost over all ring nodes.

1. Introduction

Wavelength division multiplexing (WDM) technology has the potential to satisfy the ever-increasing bandwidth needs of network users on a sustained basis. In WDM networks, nodes are equipped with *optical cross-connects* (OXCs) or *optical add–drop multiplexers* (OADMs), devices which can optically switch wavelengths, thus making it possible to establish *lightpath* connections between pairs of network nodes. The set of lightpaths defines a *logical topology*, which can be designed to optimize some performance measure for a given set of traffic demands. The logical topology design problem has been studied extensively in the literature. Typically, the traffic demands have been

* Corresponding author.
E-mail addresses: bchen@eos.ncsu.edu (B. Chen), rouskas@eos.ncsu.edu (G.N. Rouskas), dutta@eos.ncsu.edu (R. Dutta).

1573-4277/$ - see front matter © 2005 Published by Elsevier B.V.
doi:10.1016/j.osn.2005.01.002

expressed in terms of whole lightpaths, while the metric of interest has been the number of wavelengths, the congestion (maximum traffic flowing over any link), or a combination of the two. The reader is referred to [4] for a survey and classification of relevant work. Further, since the bandwidth of a single wavelength channel is (and is expected to be in the future) realistically far more than that of individual traffic needs, electronic TDM multiplexing must be applied to efficiently utilize bandwidth; but the line terminating equipment (LTE) required to do so is a costly network component. These observations give rise to the concept of *traffic grooming* [3,6,10,14], a variant of logical topology design, which is concerned with the development of techniques for combining lower speed components onto wavelengths in order to minimize network cost. For a survey and classification of relevant work, see [6].

Most studies in literature concentrate on some *aggregate* representation of the LTE cost. That is, the objective to be minimized is usually expressed as the sum, over all network nodes, of the LTE cost at each individual node. While a metric that accounts for the network-wide LTE cost is important, minimizing the total cost in the network without imposing any bound on the cost of individual nodes may result in a solution in which some nodes end up with a (very) large amount of LTE while some others end up with only a small amount of LTE. Such a solution has a number of undesirable properties. First, a node that requires a large amount of LTE may be too expensive or even impractical to deploy (e.g., due to high interconnection costs, high power consumption, or space requirements). Second, the resulting network can be highly heterogeneous in terms of the capabilities of individual nodes, making it difficult to operate and manage. Third, and more important, a solution minimizing the total LTE cost can be extremely sensitive to the assumptions regarding the traffic pattern, as previous studies [5] have demonstrated. Specifically, a solution that is optimal for a given set of traffic demands may be far away from optimal for a different such set. Since LTE involve expensive hardware devices that are difficult to move from one node to another on demand, an approach that attempts to minimize total LTE cost may not be appropriate for dimensioning a network unless the network operator has a clear picture of traffic demands far into the future *and* these traffic demands are unlikely to change substantially over the life of the network.

In this work, we consider a variant of the traffic grooming problem in which the objective is to minimize the LTE cost at the node where this cost is maximum. We believe that a Min–Max objective, such as the one we consider in this paper, is of high practical value to network designers and network operators. In particular, an approach that minimizes the maximum LTE cost at any network node is likely to be attractive because, from practical design considerations, all the network nodes are likely to be provisioned with identical equipment. Effectively, all nodes will have a cost that is dictated by the node with the maximum LTE. Such a homogeneous network is easier to operate, manage and maintain, and is likely to be less expensive than a heterogeneous one due to the economies of scale that can be achieved when all nodes are subject to identical specifications. Furthermore, such an optimization approach can be of great importance to dimensioning the network for *unknown and/or dynamic future demands*. Specifically, the network designer may solve the optimization problem for a wide range of traffic scenarios, and equip each node with an amount of LTE equal to the highest solution obtained (plus a certain *fudge factor* for making the solution future-proof). We also note that a similar approach was taken in [2] in a different context, namely for routing and wavelength assignment in the presence of converters. Specifically, an algorithm was developed for distributing a number of converters uniformly across the ring nodes rather than placing them at a single hub node; we make use of this algorithm in the heuristics we develop in this paper.

The paper is organized as follows. In Section 2, we introduce the Min–Max objective we consider. In Section 3, we present new theoretical results which demonstrate that traffic grooming with the Min–Max objective is an inherently more difficult problem than other subproblems of logical topology design, e.g., wavelength assignment. In Section 4, we present a polynomial-time algorithm for unidirectional ring networks to minimize the maximum LTE cost at any node; the algorithm consists of two distinct parts, one for traffic grooming and one for wavelength assignment. In Section 5 we extend the algorithm for bidirectional rings in which the routing of traffic is not predetermined. We present numerical results in Section 6, and we conclude the paper in Section 7.

2. Problem statement

We study the traffic grooming problem [6] in a ring $\mathcal{R}$ with N nodes. Let C be the capacity of each wavelength, expressed in units of some arbitrary rate (e.g., OC3); we will refer to parameter C as the *grooming factor*. Let W be the number of wavelengths that each fiber link in the network can support. We represent a traffic pattern by a demand matrix $T = [t^{(sd)}]$, where integer $t^{(sd)}$ denotes the number of traffic streams (each of unit demand) from node s to node d. (We allow the traffic demands to be greater than the capacity of a lightpath, i.e., it is possible that $t^{(sd)} > C$ for some s, d.) We further assume that ring nodes have electronic switching capabilities that permit them to switch traffic between wavelengths that terminate/originate locally. Given matrix T, the traffic grooming problem involves the following conceptual subproblems (SPs):

(1) *logical topology SP:* find a set R of lightpaths,
(2) *lightpath routing and wavelength assignment (RWA) SP:* solve the RWA problem on R, and
(3) *traffic routing SP:* route each traffic stream through the lightpaths in R.

We note that the first and third subproblems together constitute the grooming aspect of the problem. Also, in this context, the number W of wavelengths per fiber link is taken into consideration as a constraint rather than as a parameter to be minimized.

The Min–Max objective we consider in this paper is:

> *to minimize the maximum number F of lightpaths originating from or terminating at any node.*

In our cost model, one unit of cost is incurred for each lightpath that terminates at, or originates from, a network node. Thus, this cost metric accurately reflects the amount of LTE needed at each network node, and therefore our objective is to minimize the LTE cost at the node where it is maximum. Note also that this objective is equivalent to minimizing the *maximum nodal degree* in the logical topology. A formulation of this problem as an integer linear programming (ILP) problem can be obtained by applying simple modifications to the ILP we present in [6].

3. Complexity results

The traffic grooming problem in ring networks is NP-complete since the RWA subproblem in rings is NP-complete. In this section, we prove that the traffic grooming problem in path networks is also NP-complete. Since the RWA problem can be solved in linear time in path networks, our results demonstrate that traffic grooming with the Min–Max objective is itself an inherently difficult problem. They also prove that the traffic grooming problem with the Min–Max objective remains NP-complete in rings or other general topologies *even when* full wavelength conversion is available at the network nodes.

Let us consider a network in the form of a unidirectional path $\mathcal{P}$ with N nodes. There is a single directed fiber link from node i to node $i+1$, for each $i \in \{1, 2, \ldots, N-1\}$. An instance of the traffic grooming problem is provided by specifying a number N of nodes in the path, a traffic matrix $T = [t^{(sd)}], 1 \le s < d \le N$, a grooming factor C, a number of wavelengths W, and a goal F. The problem asks whether a valid logical topology may be formed on the path and all traffic in T routed over the lightpaths of the logical topology so that the number of incoming or outgoing lightpaths at any node in the path is less than or equal to F.

We first consider the case where bifurcated routing of traffic is not allowed. Specifically, for any source–destination pair (s, d) such that $t^{(sd)} \le C$, we require that all $t^{(sd)}$ traffic units be carried on the *same* sequence of lightpaths from source s to destination d. On the other hand, if $t^{(sd)} > C$, it is not possible to carry all the traffic on the same lightpath. In this case, we allow the traffic demand to be split into $\lfloor \frac{t^{(sd)}}{C} \rfloor$ subcomponents of magnitude C and at most one subcomponent of magnitude less than C, and the no-bifurcation requirement applies to each subcomponent independently.

Theorem 3.1. *The decision version of the grooming problem in unidirectional paths with the Min–Max objective (bifurcated routing of traffic not allowed) is NP-complete.*

Proof. The proof is provided in Appendix A.

Because of the construction in the above proof, we have the following corollary. This corollary demonstrates that, even when solutions to the first two subproblems of the traffic grooming problem (refer to Section 2) are provided, the problem remains NP-complete by virtue of the third subproblem (traffic routing). Therefore, traffic grooming is inherently more difficult than the well-known NP-complete RWA problem.

Corollary 3.1. *The decision version of the traffic grooming problem in unidirectional paths with the Min–Max objective (bifurcated routing of traffic not allowed) is NP-complete even when a logical topology is provided.*

We now extend the above results to the case where bifurcated routing of traffic is allowed. Specifically, a traffic component $t^{(sd)}$ is allowed to be split into various subcomponents which may follow different routes (i.e., different lightpath sequences for a path network) from source to destination. The bifurcation is restricted to integer subcomponents.

Theorem 3.2. *The decision version of the grooming problem in unidirectional paths with the Min–Max objective (bifurcated routing of traffic allowed) is NP-complete.*

Proof. The proof is provided in Appendix B.

Again, by the nature of the proof, we are able to state the following:

Corollary 3.2. *The decision version of the traffic grooming problem in unidirectional paths with the Min–Max objective (bifurcated routing of traffic allowed) is NP-complete even when a candidate logical topology is provided.*

Let us now consider the implications of these results for related topologies, *bidirectional* path networks, and *ring* networks (unidirectional and bidirectional). The implications for ring networks are of practical importance, even though the NP-hard nature of traffic grooming for ring networks has already been demonstrated. In particular, it is known that the RWA problem in rings is NP-hard [12]. However, the following corollaries show that *even if all ring nodes are equipped with wavelength converters* (in which case wavelength assignment is trivial), traffic grooming with the Min–Max objective remains a difficult problem. We state below three lemmas which settle the question for these topologies without proof. In each case, the proof is straightforward and can be obtained by an appropriate restriction of the traffic matrix. Specifically, to obtain an instance of a bidirectional problem from the corresponding instance of a unidirectional problem, we add traffic demands between adjacent nodes in the opposite direction, each demand equaling the full capacity of the link in the opposite direction, and we add W to the Min–Max objective F. Also, we obtain an instance of a ring problem from the corresponding path problem instance by merging the two nodes at the ends of the path into a single node, and modifying the traffic matrix appropriately.

Corollary 3.3. *The decision version of the traffic grooming problem in bidirectional path networks with the Min–Max objective (bifurcated routing of traffic allowed or not allowed) is NP-complete.*

Corollary 3.4. *The decision version of the traffic grooming problem in unidirectional ring networks with the Min–Max objective (bifurcated routing of traffic allowed or not allowed) is NP-complete, even when every node has full wavelength conversion capability.*

Corollary 3.5. *The decision version of the traffic grooming problem in bidirectional ring networks with the Min–Max objective (bifurcated routing of traffic allowed or not allowed) is NP-complete, even when every node has full wavelength conversion capability.*

Finally, the following theorem shows that, when bifurcation of traffic is allowed, the grooming problem with the Min–Max objective is not approximable in P, unless $P = \mathrm{NP}$.

Theorem 3.3. *If* $P \neq \mathrm{NP}$, *then no polynomial-time approximation algorithm* A *for the traffic grooming problem in unidirectional path networks with the Min–Max objective (bifurcated routing of traffic allowed) can guarantee* $A(\mathrm{Instance}) - OPT(\mathrm{Instance}) \leq K$ *for a fixed constant integer* K.

Proof. The proof is provided in Appendix C.

Since general network topologies, including most interesting topology families such as spiders, rings, grids, tori, contain the path network as a sub-family, the above result shows that it is not practical to attempt optimal or constant ratio approximate solutions to the grooming problem with the Min–Max objective in these cases. The only family which does not include the path as a special case is the star topology. We have considered star networks elsewhere [1], and showed that the problem is again NP-complete. Whether approximations are possible for star networks is a question that remains open at this time.

4. Min–Max traffic grooming algorithm for unidirectional rings

We now present a polynomial-time algorithm for traffic grooming in WDM ring networks with the goal of minimizing the maximum nodal degree (indegree or outdegree) in the logical topology, consistent with the Min–Max objective function. As we mentioned earlier, the degree of a node is a reflection of the LTE cost needed at that node. Rather than solving all three subproblems of the traffic grooming problem simultaneously (refer to Section 2), we decouple the logical topology and traffic routing subproblems from the RWA subproblem and tackle them independently. Specifically, our algorithm consists of the following steps:

- **Step 1.** Solve the logical topology and traffic routing subproblems on the ring network using the algorithm in Section 4.1. The result of this step is a set R of lightpaths (logical topology) and a routing of the traffic demands over the lightpaths in R that minimize the maximum amount of LTE at any node.
- **Step 2.** Use the algorithm in Section 4.2 to color the lightpaths of set R. The result of this step is a wavelength assignment that does not use more than W wavelengths. However, at the end of this step, the amount of LTE at one node, say, node i, of the ring may increase beyond the corresponding value after Step 1, by an amount equal to some value Δ.
- **Step 3.** Use the algorithm presented in [2] to distribute the additional Δ LTE at node i to other nodes in the ring network.

The following subsections explain the steps of our Min–Max traffic grooming algorithm in more detail.

4.1. Min–Max traffic grooming algorithm

In this section, we present a polynomial-time algorithm for the logical topology and traffic routing subproblems of the traffic grooming problem. Unlike previous studies, our algorithm attempts to minimize the maximum amount of LTE at any ring node by creating long lightpaths that bypass intermediate nodes whenever possible. Note that, because of the results we presented in Section 3, the traffic grooming subproblem is itself NP-complete, and hence our polynomial-time algorithm will terminate without necessarily finding an optimal solution. However, numerical results to be presented later indicate that the solutions obtained using our algorithm are close to the optimal and/or the lower bound.

Before we proceed, we introduce the concept of *reduction* of a traffic matrix. Specifically, we reduce the matrix T so that all elements are less than the capacity C of a single wavelength, by assigning a whole lightpath to traffic between a given source–destination pair that can fill it up completely. The available wavelengths on the links of the path segment from the source to the destination node are also decremented by the number of lightpaths thus assigned. Since breaking such lightpaths would increase the amount of LTE at some intermediate nodes of the path, this procedure does not preclude us from reaching an optimal solution, nor does it make the problem inherently easier or more difficult. We continue using the same notation for the traffic matrix and traffic components, but in what follows they stand for the same quantities after the reduction process.

After the reduction, we initialize the logical topology to one in which a sufficient number of single-hop lightpaths is formed on each link of the ring network to carry the traffic using this link. By *single-hop lightpath* we mean a lightpath which only traverses one physical fiber link. We note that this initial solution is a feasible solution to the logical topology and traffic routing subproblems, in that it does not use more than W lightpaths on any link. However, this initial topology yields a large value for the Min–Max objective F, which is equal to the number of single-hop lightpaths in the most congested link. Our approach, then, is to improve on this initial solution by joining short lightpaths to form longer ones, thus lowering the degrees at intermediate nodes. In the following, we summarize our algorithm for joining short lightpaths.

Let us define the relationship $i \prec j$ between ring nodes to denote that node i "precedes" node j in the direction of traffic flow; similarly, we will use the notation $i \preceq j$ to denote that node i precedes, or may be the same as, node j. The main idea of our algorithm is to consider the node with the maximum degree, and to attempt to decrease its degree by one at each iteration; this process repeats until no more improvement is possible. Let m be the node with the maximum degree. The algorithm searches for a pair of nodes $(i, j), i \prec m \prec j$, such that there exist lightpaths (i, m) and (m, j). The objective is to shift all the traffic from lightpaths (i, m) and/or (m, j) to either an existing or a new lightpath (i, j) in order to decrease the maximum of the indegree and outdegree of node m by one. If the traffic can be shifted entirely to an existing lightpath (i, j), then this procedure is always possible, since no wavelength limit constraints are violated, and also the degree of nodes i and/or j may also decrease in the process. However, if a new lightpath (i, j) must be created, the above procedure is carried out only if the wavelength limit constraint is not violated and the degrees of nodes i and j do not increase above the current maximum degree minus one (the minus one is necessary to ensure that the algorithm will not get into an infinite loop). A more detailed description of the algorithm is provided in Fig. 1.

We now argue that at the end of each iteration of the **repeat** loop, the algorithm produces a solution to the logical topology and traffic routing subproblems that is feasible (i.e., no link carries more than W lightpaths — recall that we are not concerned with wavelength assignment at this point), and the maximum nodal degree is no larger (but possibly smaller) than that of the logical topology at the beginning of the iteration. At each iteration of the **repeat** loop, the algorithm tries to replace the lightpaths (i, m) and (m, j) for some nodes $i \prec m \prec j$ with a (possibly) new and longer lightpath (i, j) so as to decrease the nodal degree of node m. However, no action is taken if replacing the two lightpaths with the longer one would violate any wavelength constraints or would increase the degrees of i or j to more than the maximum at the start of the iteration, as we can see at Step 15. Since the initial topology at Step 2 of the algorithm is feasible, we conclude that the topology at the end of each iteration will be feasible and will not increase the maximum nodal degree.

Min–Max Traffic Grooming Algorithm

Input: A ring network with N nodes, W wavelengths each of capacity C, and reduced traffic matrix $T = [t^{(sd)}]$.

Output: The number of lightpaths b_{ij} from i to j, and the traffic routing quantities $t_{ij}^{(sd)}$ (which indicate the amount of the component $t^{(sd)}$ routed over a lightpath from i to j), so that the solution does not violate any wavelength constraints and minimizes the maximum number of lightpaths terminating at, or originating from, any node.

1. **begin**
2. Initialize the logical topology to one with only single-hop lightpaths and initialize all b_{ij} and $t_{ij}^{(sd)}$ accordingly
3. **for all** i, j **do** $r_{ij} \leftarrow$ capacity unused on the (i, j) direct lightpath in the current topology (residual capacity)
4. **for all** j **do** $I_j \leftarrow$ indegree of j in the current topology
5. **for all** j **do** $O_j \leftarrow$ outdegree of j in the current topology
6. **repeat** // Main iteration
7. $m \leftarrow$ some node s. t. $\max\{I_m, O_m\}$ is maximum in the ring
//We wish to replace a lightpath (i, m) and/or (m, j) with a
//direct lightpath (i, j) so as to reduce $\max\{I_m, O_m\}$ by one
8. **for** each pair (i, j) such that $i \prec m \prec j$ **do**
9. **if** $I_m = O_m$ **then** *TotalToShift* $\leftarrow \max\{C - r_{im}, C - r_{mj}\}$
else if $I_m > O_m$ **then** *TotalToShift* $\leftarrow C - r_{im}$
else *TotalToShift* $\leftarrow C - r_{mj}$
// *TotalToShift* is the amount of traffic to be shifted to other
// wavelengths in order to reduce $\max\{I_m, O_m\}$ by one
10. *TrafficToShift* $\leftarrow 0$
11. **for** each pair (s, d), $s \preceq i$, $j \preceq d$ **do**
12. *TrafficToShift* $\leftarrow$ *TrafficToShift* $+ \min(t_{im}^{(sd)}, t_{mj}^{(sd)})$
13. **if** *TrafficToShift* $>$ *TotalToShift* **then** break
// No more (s, d) pairs needed
14. **endfor** // of the (s, d) loop
15. **if** *TrafficToShift* $<$ *TotalToShift* **then** break
// Cannot replace lightpaths (i, m) or (m, j)
// Continue w/ next (i, j) pair
else if replacing lightpaths (i, m) or (m, j) would violate any wavelength limit constraints **or** would create new maximum total degrees at i or j **then** break
else
16. Remove lightpaths (i, m), (m, j)
// $\max\{I_m, O_m\}$ decreases by 1
17. **for** all pairs (s, d) contributing to *TrafficToShift* **do**
18. Reduce $t_{im}^{(sd)}$, $t_{mj}^{(sd)}$ by contributing amount and add equal amount to $t_{ij}^{(sd)}$
// This step reflects the new routing over lightpath (i, j)
19. **endif**
20. **endfor** // of the (i, j) loop
21. **until** no decrease in the max degree at any node is possible
22. **end** // of the algorithm

Fig. 1. Algorithm for logical topology and traffic routing.

The running time complexity of our algorithm is determined by the main iteration between Steps 6 and 21. In turn, the complexity of the iteration is determined by the two **for** loops, the inner **for** loop from Step 11 to 14, and the outer loop from Step 8 to 20. Each of these loops takes time $O(N^2)$ in the worst case, where N is the number of nodes in the ring. Therefore, each iteration through the **repeat** loop from Step 6 to 21 takes $O(N^4)$ time in the worst case. The main iteration of the algorithm (i.e., the **repeat** loop) will be executed at most $N\delta$ times, where δ is the maximum decrease in the degree of any node. Since the value of δ is always less than the number W of wavelengths, the worst-case complexity of the algorithm in Fig. 1 is $O(WN^5)$. However, as we discuss in the next section, in practice, our algorithm runs much faster than the above worst-case analysis indicates; in fact it has never taken more than a few tens of milliseconds for any problem instance with $N = 16$ nodes and $W = 128$ wavelengths.

4.2. An algorithm for wavelength assignment

The output of the algorithm we presented in the previous subsection is a set of lightpaths R between pairs of ring nodes (including lightpaths assigned to traffic demands equal to the lightpath capacity before the reduction step), and a routing of the traffic elements $\{t^{(sd)}\}$ over these lightpaths. While the algorithm guarantees that the resulting logical topology is such that no link in the ring network carries more than W wavelengths, it may not be possible to color the lightpaths in R using no more than W wavelengths. In fact, the problem of deciding whether there exists a coloring of the set of lightpaths R that uses no more than W wavelengths is NP-complete [11]. In this subsection, we present a polynomial-time algorithm to perform wavelength assignment with at most W colors; the tradeoff in ensuring that the number of wavelengths does not exceed W is a modification of the logical topology (i.e., the set R) which may result in an increase in the degree of some node in the ring. Consequently, the objective F of our optimization problem may increase. Therefore, we then refine the new logical topology to decrease the objective F.

Let us start by describing how to assign wavelengths to the lightpaths of set R. Our approach is based on the observation that, while the wavelength assignment problem is hard for ring networks, it is solvable in linear time in paths [9]. Consider some node m of the ring. Let R_1 denote the lightpaths in R which *optically bypass* node m, and let $R_2 = R - R_1$ be the set of remaining lightpaths. The lightpaths in set R_2 can be viewed as the logical topology on a path network, and thus, can be colored using no more than W wavelengths. Now consider all the lightpaths in set R_1. It may be possible to color some of them without violating any wavelength continuity constraints; in general, however, there may be some lightpaths in this set that cannot be colored without the need for additional wavelengths. In this case, we break such a lightpath (x, y), $x \prec m \prec y$ into two lightpaths (x, m) and (m, y). The new lightpaths do *not* bypass node m, and thus, can be colored along with the lightpaths in set R_2 using no more than W wavelengths. While breaking such a lightpath will increase the indegree and outdegree of node m by one, this approach guarantees a coloring of the new set of lightpaths that satisfies the wavelength constraints. The following steps describe our algorithm in more detail.

(1) Let $R_1 \subset R$ be the set of lightpaths that optically bypass node m. Let $R_2 = R - R_1$ be the subset of remaining lightpaths.
(2) Sort the lightpaths in R_2 in increasing order of their length.
(3) Use the first-fit policy to color the lightpaths in R_2. Note that this step is always possible since it corresponds to a first-fit wavelength assignment for a path network.
(4) Sort the lightpaths in R_1 in decreasing order of their length.
(5) Use the first-fit policy to color the lightpaths in R_1. If lightpath $l = (x, y)$, $y \prec x$, cannot be colored, then: break l into two lightpaths, $l_1 = (x, m)$ and $l_2 = (m, y)$ which do not bypass node m; increment the indegree and outdegree of node m to accommodate the new lightpaths; and repeat from Step 1 with $R_2 \leftarrow R_2 \cup \{l_1, l_2\}$ and $R_1 \leftarrow R_1 - \{l\}$.

Let Δ denote the increase in the nodal degree of node m after the termination of the above wavelength assignment algorithm. This increase is due to the fact that Δ lightpaths which optically bypassed node m under the initial logical topology defined by the set

R, have now been broken into two lightpaths each. Of the 2Δ new lightpaths, Δ terminate at node m and Δ originate from it. Therefore, node m needs an additional Δ pairs of LTE, one for each of the original Δ lightpaths that used to bypass the node. Consequently, this increase of the objective F by Δ at node m increases the LTE cost in the ring network.

We now show how we can refine the new logical topology at the end of the wavelength assignment algorithm to improve on the objective F. Our approach is based on the observation that each additional pair of LTE at node m, one for an incoming and one for an outgoing lightpath, can be thought of as a *wavelength converter*. Indeed, consider, one of the Δ lightpaths that initially bypassed the node. This lightpath was broken by the algorithm into two shorter lightpaths that terminate at and originate from the node, respectively. This action was taken in Step 6 of the algorithm because it was not possible to assign the two shorter lightpaths on the two links in either side of node m the same color. Therefore, the additional pair of LTE at node m acts as a converter, changing the wavelength of the new incoming short lightpath to the wavelength of the new outgoing lightpath.

Consider a logical topology and corresponding feasible wavelength assignment on a ring network that requires a number Δ of converters at some node m. The recent study in [2] showed that it is possible to modify the wavelength assignment such that 2Δ converters are uniformly distributed across all N ring nodes (i.e., each node has at most $\lceil 2\Delta/N \rceil$ converters). Therefore, we use this algorithm to distribute the Δ pairs of LTE (i.e., "converters") at node m to the other ring nodes. As a result, the maximum degree at all the ring nodes will increase, but the maximum nodal degree of the network (i.e., of node m) will decrease, resulting in a new logical topology with a smaller value for the objective F. For the details of this algorithm, the reader is referred to [2].

5. Algorithm for bidirectional rings

In bidirectional rings, we assume that between every two adjacent nodes there are two links, each carrying W wavelengths, in opposite directions. The objective is to minimize the maximum in/out nodal degrees in the logical topology, regardless of the lightpath direction. Also, we make the following two assumptions. First, we *do not allow* a single lightpath to occupy both directions of a link. Second, we *do allow* a traffic component to be carried from its source node s to its destination node d on a sequence of lightpaths some of which are in one direction and some in the reverse direction; thus a traffic component may traverse the same link multiple times in either direction. Such routing of traffic components may offer advantages in terms of the Min–Max objective we consider here, if it can make use of the remaining capacities in these lightpaths.

We now present a traffic grooming heuristic for bidirectional ring networks. The heuristic first applies shortest path routing to decompose the bidirectional problem instance into two unidirectional problem instances. The heuristic then solves each unidirectional subproblem using the algorithm we presented in Fig. 1, and combines the individual solutions into a solution for the original problem by adding the in/out nodal degrees determined by each solution for each node. Note that, if the subproblems are solved independently of each other, it may happen that some node x be the maximum degree node in both solutions. In this case, the final combined solution may not be a good one, since node x may end up with a large overall degree. Therefore, we modify the algorithm to take into account the overall objective (for the original bidirectional problem). In this approach, the solution to each unidirectional subproblem takes into account the solution to the other subproblem, and vice versa. Specifically, our bidirectional traffic grooming algorithm consists of the following steps.

- **Step 1.** Use shortest path routing to determine the direction in which each traffic component will be routed. Decompose the original traffic matrix T into two traffic matrices T_r and T_l containing the components routed in the clockwise and counterclockwise direction, respectively, and such that $T = T_r + T_l$. This decomposition creates two unidirectional ring subproblems with respective traffic matrices T_r and T_l.
- **Step 2.** Use the heuristic we developed in the previous section (refer to Fig. 1) to solve the clockwise ring, and record the resulting in/out degrees at each node. Then, solve the counterclockwise ring, with the following minor

change in the heuristic: when selecting the node m with the maximum degree (refer to Step 7 in Fig. 1), we add the current degrees for both directions, so that m is the node with the maximum *aggregate* degree. At the end of this step, we obtain a solution S_1.

- **Step 3.** Repeat Step 2, but this time solve the counterclockwise ring first, then solve the clockwise ring accordingly, to obtain a solution S_2. Return the best solution among S_1 and S_2.

It is clear that the asymptotic complexity of the above algorithm is the same as that of the unidirectional ring algorithm.

6. Numerical results

In this section, we present experiments to demonstrate the performance of our traffic grooming algorithm. The experiments are characterized by the following parameters: the traffic pattern, the number N of nodes in the ring, the number W of wavelengths per link, the capacity C of each wavelength, and the load L on the link carrying the most traffic. The maximum amount of traffic that can flow through a link is WC; hence we express the load L as a percentage of WC. For each experiment (i.e., each set of values for the above parameters), we generate 50 problem instances. We consider three traffic patterns in our study. For each traffic pattern, the traffic matrix $T = [t^{(sd)}]$ of each of the 50 problem instances is generated by drawing $N(N-1)$ random numbers (rounded to the nearest integer) from a Gaussian distribution with a given mean t and standard deviation σ that depend on the traffic pattern. The three traffic patterns are:

(1) *Uniform pattern.* To generate this traffic pattern, we first determine the mean value t of the Gaussian distribution according to the desired load L, and we let the standard deviation be 10% of the mean t.

(2) *Random pattern.* For this pattern, we let the standard deviation of the Gaussian distribution be 150% of the mean t. Consequently, the traffic elements $t^{(sd)}$ take values in a wide range around the mean, and the loads of individual links also vary widely. If the random number generator returns a negative value for some traffic element, we set the corresponding $t^{(sd)}$ value to zero. Also, if a traffic matrix generated in this manner is infeasible (i.e., the load on some link exceeds the value WC), then we discard it and we generate a new matrix for the corresponding problem instance.

(3) *Locality pattern.* This traffic pattern is designed to capture the traffic locality property that has been observed in some networks. Specifically, the traffic elements $t^{(sd)}$ are generated such that, on average, 50% of the traffic sourced by any node s is destined to the node one hop away, 30% is for the node two hops away, and 10% is for the node three hops away. The remaining 10% of the traffic generated by node s is distributed equally among the other $N-4$ nodes of the ring.

6.1. *Results for small unidirectional ring networks*

We were able to use to CPLEX to solve the ILP for the traffic grooming problem only for ring networks with up to $N = 8$ nodes. We now present experiments with eight-node rings in order to compare the results of our algorithm to the optimal solution. In the figures shown in this section, we compare the following values for each instance:

- The *lower bound* F^l on the objective F given by

$$F^l = \max_s \left(\max \left(\left\lceil \frac{\sum_d t^{(sd)}}{C} \right\rceil, \left\lceil \frac{\sum_d t^{(ds)}}{C} \right\rceil \right) \right). \tag{1}$$

  To obtain this lower bound, we note that each node must source and terminate a sufficient number of lightpaths to carry the traffic demands from and to this node, respectively.
- The *optimal value* $F^\star$ of the objective F, obtained by using CPLEX to solve the ILP.
- The value of the objective F returned by our algorithm.
- The value F^e of the objective for a network using *all-electronic routing*. This is the value of the objective for a logical topology that consists of only single-hop lightpaths, i.e., one in which no optical switching of wavelengths takes place. The value of F^e corresponds to the number of wavelengths needed to carry the traffic on the link with the heaviest traffic load.

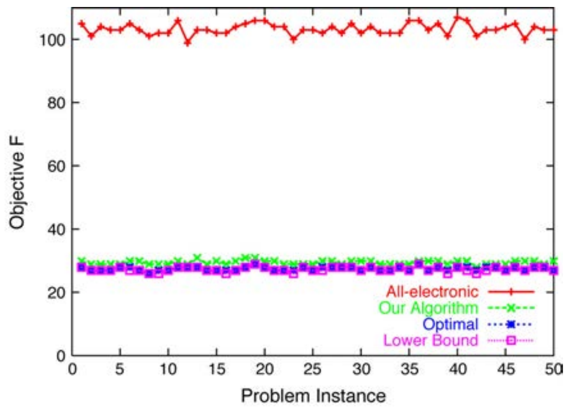

Fig. 2. Uniform pattern, $N = 8$, $W = 128$, $C = 12$, $L = 80\%$.

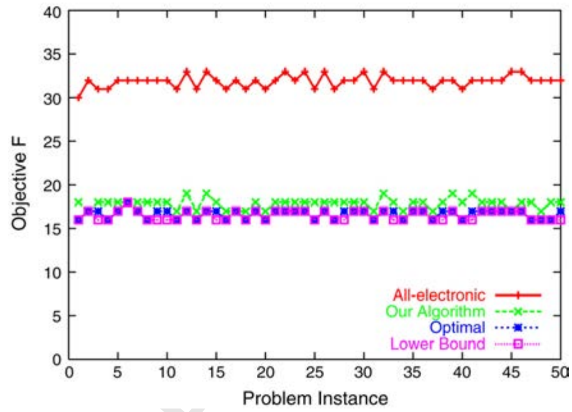

Fig. 3. Locality pattern, $N = 8$, $W = 64$, $C = 12$, $L = 50\%$.

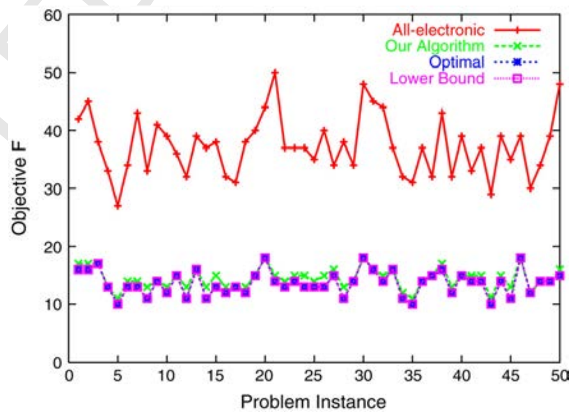

Fig. 4. Random pattern, $N = 8$, $W = 64$, $C = 12$, $L = 50$–80%.

Figs. 2–4 plot the lower bound F^l, the optimal value $F^\star$, the value of the goal F returned by our algorithm, and the all-electronic value F^e, for 50 instances of each experiment, and for the stated values of parameters W, C, and L. Fig. 2 shows results for the uniform pattern, Fig. 3 for the locality pattern, and Fig. 4 for the random pattern. We first observe that the value of the objective F returned by our algorithm is close to the optimal, and that the optimal is close to the lower bound. On the other hand, the value of the all-electronic solution is significantly higher than the other three values, anywhere from twice to four times larger. This relative behavior of the four curves is consistent across all three traffic patterns, and has been observed for a wide range of values for the system parameters. These results demonstrate the effectiveness of our algorithm, which can find a solution close to the optimal in a tiny fraction of the time required for CPLEX to terminate; in fact, for the experiments presented in this section, the running time of our algorithm never exceeds a few milliseconds, whereas CPLEX, depending on the values of the parameters, may take anywhere from a few minutes to a few hours.

We can also see that the value F^e for the all-electronic solution is always close to LW: since the traffic load on the most congested link is LWC, the number of lightpaths needed to carry this traffic is $\lceil LW \rceil$, and these lightpaths must terminate at the nodes at the two ends of this link, requiring an equal amount of LTE at each node. On the other hand, the solution returned by our algorithm, as well as the optimal one, are significantly lower than the all-electronic value. This result indicates that our approach of minimizing the maximum LTE cost in the network can produce significant cost savings. Another related observation is that, by using a Min–Max objective, we can ensure that the cost of any individual network node is determined by the traffic demands of the node, and will not scale with the number of wavelengths. Specifically, the values of the objective F in Figs. 2–4 are small compared to the number of wavelengths we used for the experiments ($W = 64, 128$), and are close to the lower bound, which represents the minimum LTE cost to accommodate the traffic demands for any ring node (see (1)).

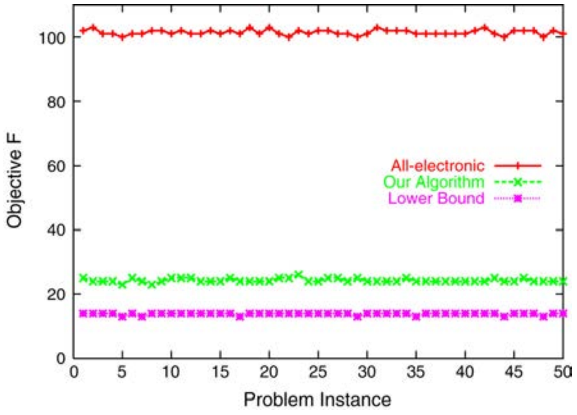

Fig. 5. Uniform pattern, $N = 16$, $W = 128$, $C = 12$, $L = 80\%$.

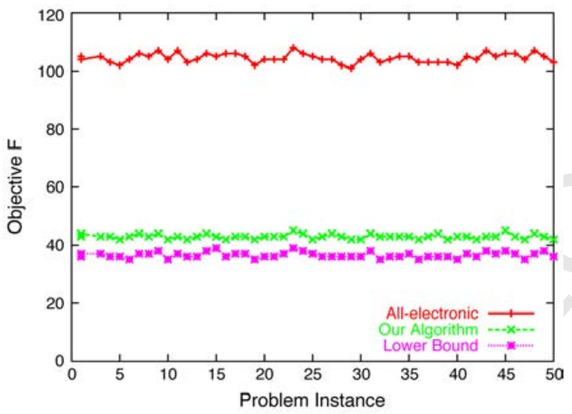

Fig. 6. Locality pattern, $N = 16$, $W = 128$, $C = 12$, $L = 80\%$.

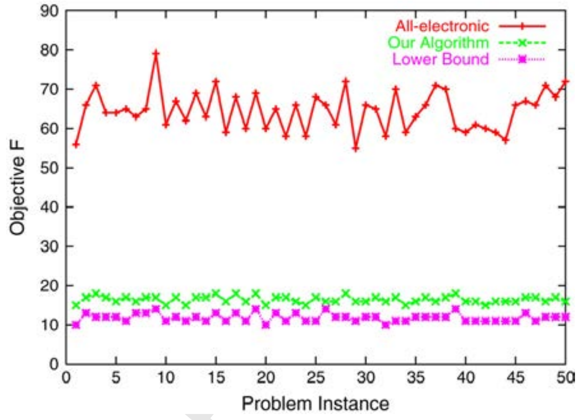

Fig. 7. Random pattern, $N = 16$, $W = 128$, $C = 12$, $L = 40-60\%$.

6.2. Results for large unidirectional ring networks

In this section, we present results for ring networks with $N = 16$ nodes and $W = 128$ wavelengths. Since we were not able to use CPLEX to obtain the optimal solution, the figures in this section only contain three curves: one for the all-electronic value F^e, one for the value of F returned by our algorithm, and one for the lower bound F^l. Our algorithm, on the other hand, needed less than a second to find a solution for each problem instance that we present in this section.

Figs. 5–7 present the results of 50 different problem instances for each of the uniform, locality, and random traffic patterns, respectively. The relative behavior of the three curves in these figures is very similar to that we observed in Figs. 2–4. In particular, the values of the objective F returned by our algorithm are close to the lower bound, and track it well across the different traffic patterns and problem instances within each pattern. The all-electronic solutions, on the other hand, are again quite high with values at around LW. Therefore, all the conclusions regarding the scalability and cost-effectiveness of our traffic grooming approach are valid for large WDM ring networks as well.

6.3. The effect of the traffic load

Let us now investigate how our solutions scale with the load L on the most congested link of the network. In Fig. 8, we plot the values of F^e, F^l, and F against the load L for a uniform traffic pattern; each point in the graphs is the average over 50 problem instances for the given value of L. We have obtained similar results for the other two patterns as well.

As we can see, F^e, F^l, and F all increase linearly with the load L. However, the curve corresponding to the all-electronic solution has a slope much steeper than that of the curves corresponding to the lower bound and our solution. This behavior demonstrates that the cost benefit of our optimization approach increases with the load of the network. This property is an important one, and it implies that network operators will be able to operate the network at high loads with only an incremental increase in cost.

6.4. Results for bidirectional ring networks

We now present results for bidirectional rings with $N = 5$ nodes. We have found that, due to the

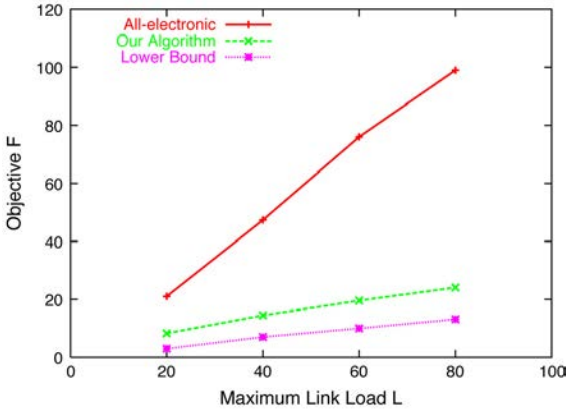

Fig. 8. Uniform pattern, $N = 16$, $W = 128$, $C = 12$, various loads.

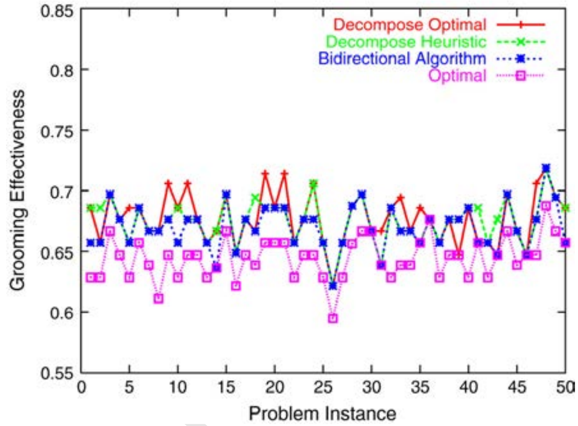

Fig. 9. Bidirectional uniform, $N = 5$, $W = 64$, $C = 12$, $F^e =$ 33–37.

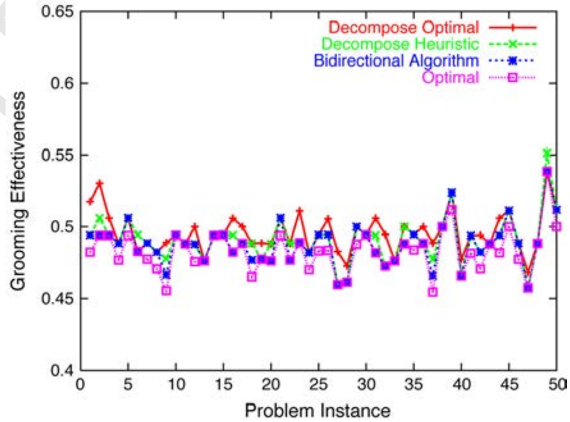

Fig. 10. Bidirectional locality, $N = 5$, $W = 64$, $C = 12$, $F^e =$ 78–94.

fact that the ILP formulation for bidirectional rings contains far more variables and constraints than in the unidirectional case, for rings with more than five nodes, CPLEX takes too long (several days) to find an optimal solution for rings with more than five nodes. We also note that, since the routing of traffic is part of the problem, it is not possible to a priori determine the loading of each link from the traffic matrix as in the unidirectional case. Therefore, we use the concept of *grooming effectiveness* to characterize the results. The grooming effectiveness is defined as the ratio of the maximum nodal degree of a solution to the one required by an all-electronic solution with shortest path routing. By definition, the smaller the grooming effectiveness, the better the solution in terms of the Min–Max objective.

For each experiment, we again generate 50 different problem instances, and we compare the traffic grooming effectiveness values for the following four values:

- The *optimal* solution returned by CPLEX.
- The *decompose optimal* solution. In this case, we use the shortest-path decomposition we described in Section 5 and we solve the two unidirectional ring instances independently using CPLEX on the corresponding unidirectional ILPs. We then combine the two solutions, and compute the overall Min–Max result (and corresponding grooming effectiveness value).
- The *decompose heuristic* solution. This is similar to the decompose optimal method, except that we use the unidirectional heuristic in Fig. 1 instead of CPLEX to solve each unidirectional ring instance.
- The *bidirectional algorithm* solution, which is obtained by running the algorithm we presented in Section 5. Recall that this algorithm focuses on the overall objective for the bidirectional ring as a whole, and is such that the solution to each unidirectional ring instance takes into account the solution to the other.

Figs. 9–11 present the grooming effectiveness values of 50 instances for each of the uniform, locality, and random traffic patterns, respectively. It is not surprising to see that, overall, the grooming effectiveness is best for the locality pattern, since the

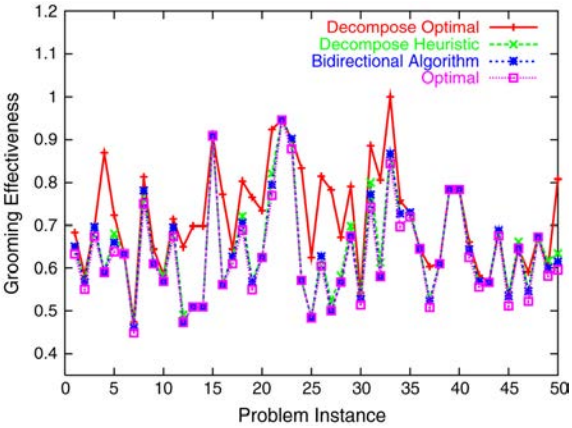

Fig. 11. Bidirectional random, $N = 5$, $W = 64$, $C = 12$, $F^e = 31$–78.

pattern itself favors shortest path routing. For both the uniform and locality patterns, all four solution approaches give similar results. However, for the random pattern, we can see that the decompose optimal method performs the worst, whereas the decompose heuristic approach performs well. This behavior may be due to the fact that CPLEX stops after finding the first optimal solution for each of the unidirectional ring instances, which may be such that maximum degree exists in the same node; our algorithm, on the other hand, returns solutions in which the nodal degrees are more balanced. Finally, the bidirectional algorithm is always close to the optimal, as expected, since it takes into consideration the overall objective.

From the three figures, we observe that our shortest-path decomposition always works well for the five-node ring networks we have considered. To further evaluate the effectiveness of the decomposition, we conducted additional experiments on five-node bidirectional rings with the random traffic pattern. We randomly generated 100,000 problem instances and obtained the optimal solutions using CPLEX. The resulting maximum nodal degrees range from 9 to 74, with an average around 33. We then run the bidirectional heuristic and recorded the difference in maximum nodal degree from the optimal solution. Of the 100,000 instances, our algorithm found the optimal solution in 38,895 cases (39%); in 55,519 cases (55.5%) our algorithm found a solution whose maximum degree is one more than the optimal; in 5,550 cases (5.5%) our solution are two more than the optimal; and in 36 cases, our solution are three more than the optimal. These results indicate that using shortest path routing in bidirectional rings works well, at least for small rings and for a wide range of traffic patterns.

6.5. *The aggregate LTE cost*

The algorithm we presented in Fig. 1 terminates once it is determined that no further improvement (reduction) of the maximum nodal degree is possible. The resulting solution has a low maximum nodal degree, but it may be such that the sum of the degrees over all network nodes is relatively high. Since the sum of nodal degrees represents the total amount of LTE that will need to be deployed in the network, it is desirable to obtain solutions with a low aggregate LTE cost. We now show how to extend the algorithm in Fig. 1 to reduce the network-wide sum of nodal degrees without increasing the maximum degree.

Specifically, rather than terminating after minimizing the maximum nodal degree, the algorithm continues to the main iteration between Steps 6 and 21. The main difference is in the manner in which the node m is selected in Step 7 and in the criterion for termination in Step 21. Note that, although it may not be possible to reduce the degree at the node where it is maximum, it may be possible to reduce the degree at some other ring node; doing so will not affect the Min–Max objective, but will reduce the aggregate nodal degree. Therefore, the algorithm uses a greedy approach, and selects the ring node m whose degree can be reduced by the maximum amount, and performs an additional iteration to reduce m's degree, taking care *not* to increase the maximum degree. This process is repeated until no further progress can be made in reducing the degree of any node in the network. Hence, this version of the algorithm achieves two objectives: the primary objective is to minimize the maximum nodal degree, and the secondary objective is to minimize the total degree over all network nodes.

In order to characterize the performance of the new algorithm with respect to the two objectives, we have conducted experiments with 16-node unidirectional rings. For each set of parameters we generated 50

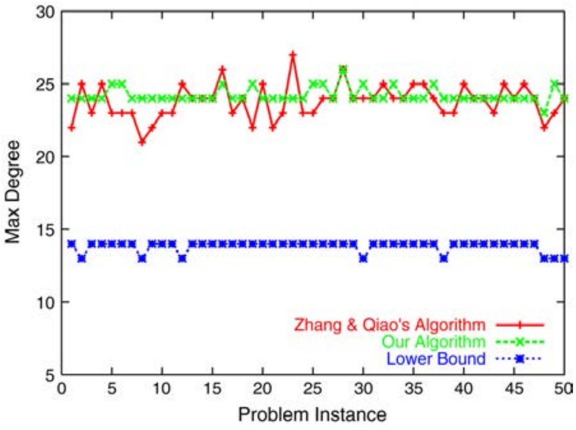

Fig. 12. Uniform, $N = 16$, $W = 128$, $C = 12$, $L = 80\%$, maximum degree.

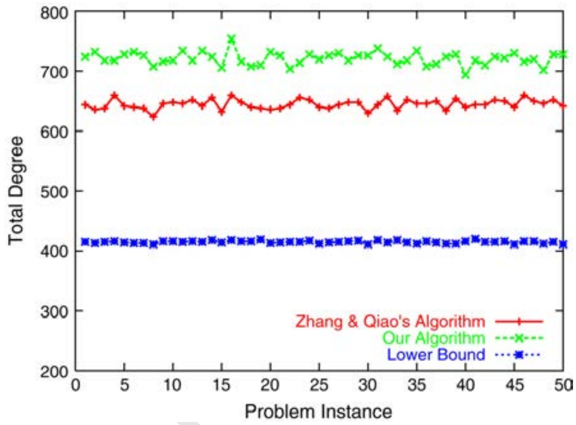

Fig. 14. Uniform, $N = 16$, $W = 128$, $C = 12$, $L = 80\%$, total degree.

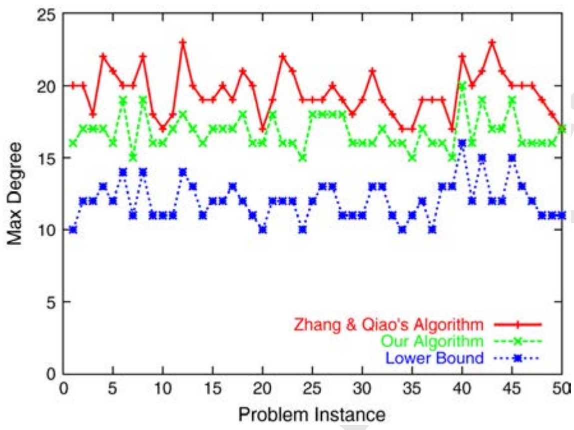

Fig. 13. Random, $N = 16$, $W = 128$, $C = 12$, $L = 80\%$, maximum degree.

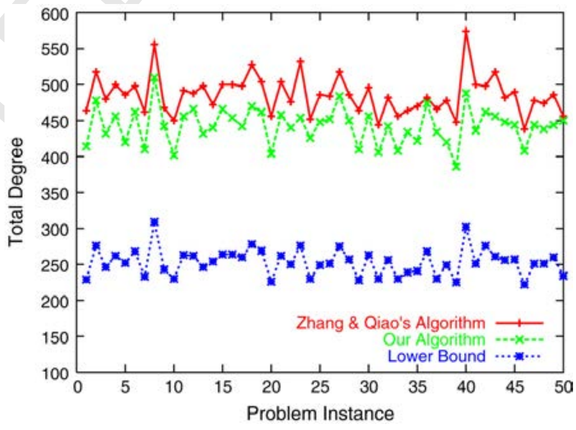

Fig. 15. Random, $N = 16$, $W = 128$, $C = 12$, $L = 80\%$, total degree.

problem instances, and we recorded the maximum nodal degree and the total degree over all network nodes for two algorithms: the algorithm in Fig. 1 after making the modifications we described above, and the algorithm by Zhang and Qiao [13]. The latter algorithm was developed specifically for minimizing the total number of SONET ADMs (i.e., the total degree) in the network. For each problem instance, we also computed the lower bound on the maximum and total degrees.

Figs. 12 and 13 plot the maximum nodal degree of each problem instance for the uniform and random traffic patterns, respectively; the results for the locality pattern are similar to those for the random pattern and are omitted. Our algorithm outperforms Zhang and Qiao's algorithm for the random (and locality) patterns, while the results for the uniform pattern are mixed, with each algorithm performing better than the other over some of the problem instances. Interestingly, the relative behavior of the two algorithms is similar in terms of the total degree (over all network nodes), as shown in Figs. 14 and 15. Again, our algorithm outperforms Zhang and Qiao's algorithm for the random (and locality) patterns, while the latter performs better for the uniform pattern. The fact that Zhang and Qiao's algorithm performs well for uniform traffic is due to the nature of the algorithm which attempts to form full circles of

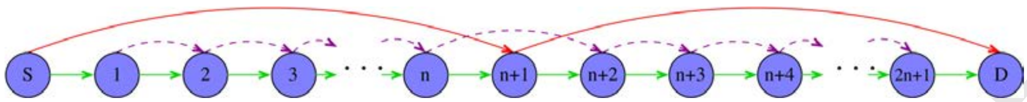

Fig. 16. Example of path construction for the proof of Theorem 3.1, $N = 2n + 3$, $W = 3$.

unit traffic components which are then groomed into wavelengths; this operation is most successful when demands are symmetric. However, for the asymmetric traffic scenarios that are more likely to be encountered in practice, our algorithm performs better, not only in terms of the maximum nodal degree, but also in terms of the total degree over all network nodes.

Overall, the results we presented in this section demonstrate that our Min–Max optimization approach for traffic grooming is successful in obtaining solutions that keep the maximum nodal degree low. Moreover, our solutions also tend to keep the overall network cost (in terms of the total degree over all network nodes) low, and compare favorably to solutions obtained by algorithms whose main objective is network-wide cost minimization.

7. Concluding remarks

We have studied a new variant of the traffic grooming problem in ring networks, where the objective is to minimize the maximum LTE cost at any node. We have developed new algorithms for traffic grooming and wavelength assignment in ring networks. We have demonstrated that our algorithms perform well for a wide range of traffic patterns and system parameter values, not only in terms of the maximum LTE cost, but also in terms of the total LTE cost over all network nodes.

Appendix A. Proof of Theorem 3.1

The reduction is from the Subset Sum problem [8]. An instance of the Subset Sum problem consists of n elements of size $w_i \in Z^+, \forall i \in \{1, 2, \ldots, n\}$, and a goal B. The question is whether there exists a subset of elements whose sizes total B. Let $B_1 = \max\{B, \sum_i w_i - B\}$. Construct a path network using the following transformation: $N = 2n + 3$, $W = 3$, $C = \sum_i w_i + 1$, and let the objective be $F = 2$. The $2n + 3$ nodes of the path are labeled $S, 1, 2, \ldots, 2n + 1, D$, and the traffic matrix is

$$t^{(sd)} = \begin{cases} C + 1, & s \in \{1, 2, \ldots, n-1\} \\ & \cup\{n+2, n+3, \ldots, 2n\}, \\ & d = s + 1; \\ B_1 + 1, & s = n, d = n + 1; \text{ or} \\ & s = n + 1, d = n + 2; \\ C - B_1, & s = n, d = n + 2; \\ w_s, & s \in \{1, 2, \ldots, n\}, \\ & d = s + (n + 1); \\ C, & s = S, d = n + 1; \text{ or } s = n + 1, \\ & d = D; \\ C, & s = S, d = 1; \text{ or } s = 2n + 1, \\ & d = D \\ 0, & \text{otherwise.} \end{cases}$$

The traffic matrix is set up so that we are forced to use the virtual topology shown in Fig. 16. Specifically, the first of the three wavelengths is used to form two lightpaths, one from node S to node $n + 1$, and one from node $n + 1$ to node D; both these lightpaths are filled to capacity carrying the corresponding traffic demands of magnitude C. Therefore, this first wavelength cannot be used to carry any other traffic. The second wavelength is used to form single-hop lightpaths between adjacent pairs of nodes in the path; these lightpaths are denoted by the straight arrows in Fig. 16. However, note that the traffic demands in the top row of the traffic matrix above are greater than the capacity C of a wavelength. Therefore, the third wavelength is used to form single-hop lightpaths between nodes $(i, i + 1), i = 1, \ldots, n - 1, n + 2, \ldots, 2n$, in the path. The single-hop lightpaths on this third wavelength between any such pair $(i, i + 1)$ of nodes carry the following traffic components: (1) the one unit of traffic from node i to node $i + 1$ (the other C units of such traffic are carried by the single-hop lightpath on the second wavelength), and (2) the w_j units of traffic from node j, $j = 1, \ldots, i$, to node $j+n+1$. Since $C = \sum_i w_i + 1$, the single-hop lightpath on the third wavelength from node i to $i + 1$ has enough capacity to carry this traffic, for all such i. Finally, the third wavelength is also used to form a

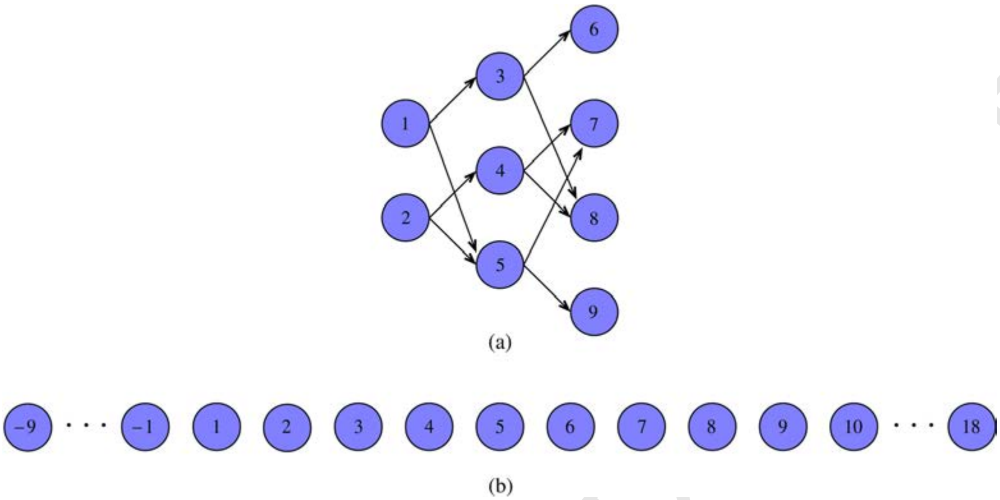

Fig. 17. Example of path construction for the proof of Theorem 3.2: (a) the original MCF instance, and (b) the corresponding path network.

direct lightpath from node n to node $n+2$ that bypasses node $n+1$, and the result is the virtual topology in Fig. 16. Note that no more lightpaths can be added to this virtual topology, and no existing lightpaths can be split without violating the objective F.

After grooming all the traffic except for w_i, which have to go through node $n+1$ with or without stopping, the capacity left in the lightpath from node n to node $n+2$ is equal to B_1, and the capacities left in the lightpaths from node n to node $n+1$ and from node $n+1$ to node $n+2$ are both equal to $\sum_i w_i - B_1$. Since bifurcation is *not* allowed, it is possible to use the virtual topology above to groom the traffic, *if and only if* there is a subset of $\{t^{(i,i+n+1)}\}$ whose sum is exactly B_1. Since deciding the satisfiability of the Subset Sum problem is NP-complete, then the new grooming problem is also NP-complete. □

Appendix B. Proof of Theorem 3.2

The reduction is from the constrained multicommodity flow (MCF) problem in three-stage networks with three nodes in the second stage, which is NP-complete [7]. An instance of the constrained MCF problem has N_1, N_2, and N_3 as the sets of nodes forming the three stages, with $|N_1 \cup N_2 \cup N_3| = n$, and $|N_2| = 3$ (refer to Fig. 17(a)). $E \subset (N_1 \times N_2) \cup (N_2 \times N_3)$ is the set of edges in the network, each of unit capacity, and $Q \subset (N_1 \times N_3)$ is the set of traffic demands, each also of unit magnitude. The problem is whether a feasible flow assignment satisfying the flow constraints exists.

To transform an instance of the MCF problem to an instance of the path grooming problem, we construct a path network with $N = 3 \times n$ nodes, labeled $-n, \ldots, -1, 1, \ldots, n, n+1, \ldots, 2n$ (see Fig. 17(b)). Let I_j and O_j denote the indegree and outdegree, respectively, of node j of the three-stage network; for example $I_6 = 1$ and $O_6 = 0$ for the network of Fig. 17(a). Our decision goal is set to $F = \max_j\{I_j, O_j\}$; for the instance of Fig. 17, $F = 2$. We let the wavelength capacity be $C = 2$, and we do not impose any constraint on the number W of wavelengths; in other words, W can be as large as needed (for practical purposes, we can let $W = KN^2$, where K is an appropriate constant, so that the number of wavelengths is sufficient for setting up direct lightpaths between any pair of nodes in the path). The traffic matrix $t^{(sd)}$ for the path grooming instance is

$$t^{(sd)} = \begin{cases} 1, & (s,d) \in E \cup Q; \\ C \times (F - I_d), & \forall d = 1, 2, \ldots, n, \\ & s = d - (n+1); \\ C \times (F - O_s), & \forall s = 1, 2, \ldots, n, \\ & d = s + n. \end{cases}$$

Because of the demands of size $C(=2)$, a direct lightpath needs to be formed for each such demand in order not to exceed the goal F; the resulting logical

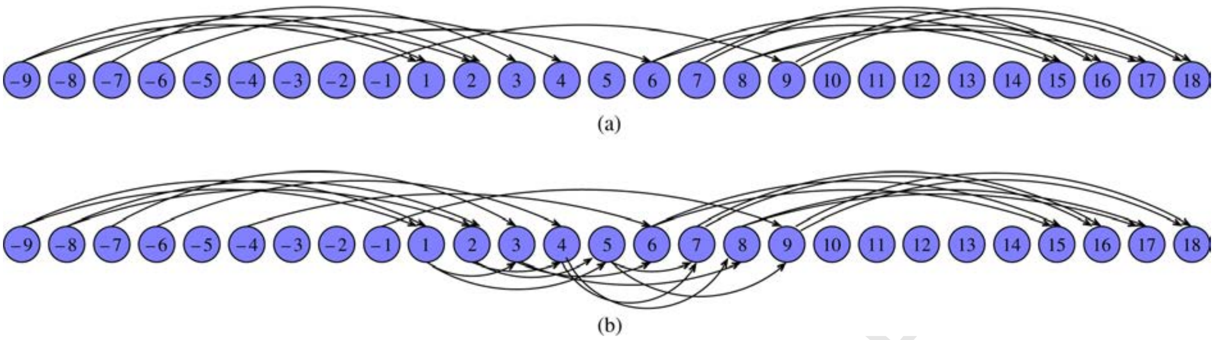

Fig. 18. The logical topology for the proof of Theorem 3.2: (a) the topology after forming lightpaths for each demand of size C, and (b) the topology after forming additional lightpaths for each demand between node pairs in set E.

topology for the example of Fig. 17(a) is shown in Fig. 18(a). Recall that $I_6 = 1$ and $O_6 = 0$ in the three-stage network of Fig. 17. Therefore, the above traffic matrix specifies $t^{(-4,6)} = C = 2$ and $t^{(6,15)} = 2C = 4$. There is one lightpath from node 4 to node 6, and this lightpath carries the total demand $t^{(-4,6)}$, while there are two lightpaths from node 6 to node 15, each of them carrying C units of the demand $t^{(6,15)}$.

The only way to satisfy the unit demands between node pairs in set E in the above traffic matrix, is to form a direct lightpath for each such demand. The resulting topology is shown in Fig. 18(b). For example, consider the source–destination pair $(1, 3) \in E$ (refer to Fig. 17(a)). A direct lightpath has been formed between nodes 1 and 3 in Fig. 18(b); similarly for other node pairs in set E.

The logical topology in Fig. 18(b) is such that exactly two lightpaths originate from, and terminate at, each node $i = 1, 2, \ldots, n$, of the path. Hence, it is not possible to add any new lightpath with such a node as the origin or termination point without violating the goal $F = 2$. Consequently, we have to use the remaining capacities left on the lightpaths formed between node pairs in set E in order to carry the demands in the above traffic matrix due to node pairs in set Q. This can be done *if and only if* the original constrained MCF problem has a solution. Since MCF is NP-complete, so is our problem. □

Appendix C. Proof of Theorem 3.3

The reduction is from the same MCF problem used above. For a given MCF problem instance, let P denote the corresponding instance of the path problem constructed in the proof of Theorem 3.2, and let F denote the corresponding objective, $F = \max_j\{I_j, O_j\}$. For this proof, we construct a path instance P' and objective F' by modifying the instance P and objective F of the previous proof as follows.

Case (a). If F exists in N_2, then we create K replicas of each node in N_1 and N_3, and connect them to N_2 in exactly the same way as the original nodes do. Let $N_1^1, \ldots, N_1^K$ denote the K replicas of node-set N_1, and, similarly, $N_3^1, \ldots, N_3^K$ denote the replicas of node-set N_3. If a node m in N_1 is connected to each node of a subset N_2' of the node-set N_2, we make the corresponding nodes $m^1, \ldots, m^K$ connect to the same nodes N_2'. We do the same for the new sets $N_3^1, \ldots, N_3^K$. We also replicate the demands between N_2 and other nodes accordingly. Also, if there is demand from node $s \in N_1$ to node $d \in N_3$, we add equal demands from each node $s^i \in N_1^i$ to each node $d^i \in N_3^i$, $i = 1, \ldots, K$. At the end of these steps, we have $K + 1$ copies of the original MCF problem, an we set the new goal $F' = (K + 1)F$. We also keep $C = 2$, and we use the same procedure as in the previous proof to transform the MCF instance to a unidirectional path problem instance, which now has $N = 3 \times (K|N_1 + N_3| + |N_2|)$ nodes. Using this polynomial-time transformation, we obtain a new traffic grooming problem P'.

Now suppose there exists a polynomial-time algorithm A that can guarantee $A(P') - OPT(P') \le K$. We solve P' using this algorithm. From the result, we can easily decide if $OPT(P) \le F$, because of the following observations:

(1) $OPT(P') = (K+1)OPT(P)$, so $OPT(P')$ must be an integer multiple of $K+1$.
(2) $A(P') - OPT(P') \leq K$; hence $OPT(P') = Y$, where Y is the largest multiple of $k+1$ less than $A(P')$.
(3) Since $OPT(P) = OPT(P')/(K+1)$, we can immediately compute $OPT(P)$ and decide if $OPT(P) \leq F$.

For any instance of MCF, we can construct an instance P' in polynomial time, use the method above to decide if $OPT(P) \leq F$ in polynomial time, then further decide if MCF has a solution, i.e., solve MCF in polynomial time. This is only possible if $P = \text{NP}$.

Case (b). If F exists in N_1 or N_3, we make K replicas of the node-set N_2, that is, we add node-sets $N_2^1, \ldots, N_2^K$, each with three nodes. We also replicate the traffic demands accordingly, and set the objective $F' = (K+1)F$. By proceeding as in Case (a), we obtain the same result.

Case (c). If F exists in both N_2 and in N_1 or N_3, use either Case (a) or Case (b) for the proof. □

References

[1] B. Chen, R. Dutta, G.N. Rouskas, Traffic grooming in star networks, in: Proceedings of the First Traffic Grooming Workshop, October 2004.
[2] L.-W. Chen, E. Modiano, Efficient routing and wavelength assignment for reconfigurable WDM networks with wavelength converters, in: Proceedings of IEEE INFOCOM, 2003.
[3] T. Cinkler, Traffic and λ grooming, IEEE Network 17 (2) (2003) 16–21.
[4] R. Dutta, G.N. Rouskas, A survey of virtual topology design algorithms for wavelength routed optical networks, Optical Networks 1 (1) (2000) 73–89.
[5] R. Dutta, G.N. Rouskas, On optimal traffic grooming in WDM rings, IEEE Journal on Selected Areas in Communications 20 (1) (2002) 110–121.
[6] R. Dutta, G.N. Rouskas, Traffic grooming in WDM networks: Past and future, IEEE Network 16 (6) (2002) 46–56.
[7] E.S. Elmallah, J.C. Culberson, Multicommodity flows in simple multistage networks, Networks 25 (1995) 19–30.
[8] M.R. Garey, D.S. Johnson, Computers and Intractability, W. H. Freeman and Co., New York, 1979.
[9] U. Gupta, D. Lee, J. Leung, Efficient algorithms for interval graphs and circular-arc graphs, Networks 12 (1982) 459–467.
[10] E. Modiano, P.J. Lin, Traffic grooming in WDM networks, IEEE Communications 39 (7) (2001) 124–129.
[11] A. Tucker, Coloring a family of circular-arc graphs, SIAM Journal of Applied Mathematics 29 (1975) 493–502.
[12] G. Wilfong, P. Winkler, Ring routing and wavelength translation, in: Proceedings of the 9th Annual ACM-SIAM Symposium on Discrete Algorithms, 1998, pp. 333–341.
[13] X. Zhang, C. Qiao, An effective and comprehensive approach to traffic grooming and wavelength assignment in SONET/WDM rings, IEEE/ACM Transactions on Networking 8 (5) (2000) 608–617.
[14] K. Zhu, B. Mukherjee, A review of traffic grooming in WDM optical networks: Architectures and challenges, Optical Networks 4 (2) (2003) 55–64.