RWA in WDM Rings: An Efficient Formulation Based on Maximal Independent Set Decomposition

Emre Yetginer
Tubitak UEKAE
Ilhan Tan Kislasi
Umitkoy, Ankara 06800 Turkey
Email: emre.yetginer@iltaren.tubitak.gov.tr

Zeyu Liu, George N. Rouskas
Department of Computer Science
North Carolina State University
Raleigh, NC 27695-8206 USA
Email: {zeyu_liu,rouskas}@ncsu.edu

***Abstract*—WDM rings are now capable of supporting more than 100 wavelengths over a single fiber. Conventional link and path formulations for the RWA problem are inefficient due to the inherent symmetry in wavelength assignment and the fact that the problem size increases fast with the number of wavelengths. Although a formulation based on maximal independent sets (MIS) does not have these drawbacks, it suffers from the exponential growth in the number of variables with the increasing network size. We develop a new ILP formulation based on the idea of partitioning the path set and representing the maximal independent sets in the original network using the independent sets calculated in each of these partitions. This formulation trades off the number of variables with the number of constraints and, as a result, achieves a much better scalability in terms of network dimension. The proposed approach is compared with existing formulations on ring networks of various sizes and it is demonstrated that the new formulation achieves more than two orders of magnitude decrease in running time, making it possible to (1) solve optimally large network instances for any number of wavelengths, which cannot be solved with classical formulations, and (2) perform extensive "what-if" analysis to evaluate the sensitivity of the optimal solutions to uncertainties in forecast traffic scenarios.**

I. Introduction

Wavelength division multiplexing (WDM) enables optical networks to divide the enormous bandwidth of an optical fiber into non-overlapping wavelength channels, which can be operated in parallel. Hence, wavelength routed optical networking has been considered as a promising approach for the realization of next generation large bandwidth networks.

In wavelength routed WDM networks, traffic is carried over optical paths (*lightpaths*) between source and destination nodes. In the absence of wavelength converters, a lightpath occupies the same wavelength channel on all the fiber links along its path and it is optically switched at intermediate nodes. The routing and wavelength assignment (RWA) is the problem of selecting a path and wavelength for each of the given connection demands, subject to the constraint that no two paths sharing a link are assigned the same wavelength.

Static RWA is one of the central problems in the dimensioning of WDM networks, and it also appears as a subproblem in many important network design applications, including traffic grooming [1]–[3], survivability design [4], [5], and traffic scheduling [6], [7]. In the static RWA problem [8], the input typically consists of a set of (forecast) traffic demands (i.e., requested connections), and the objective is either to establish all the connections using a minimum number of wavelengths, or to maximize the number of accepted connections (in which case the number of wavelengths is taken as a constraint). We refer to the former static RWA variant as the *minRWA problem*, and to the latter as the *maxRWA problem*. Both variants have been studied extensively in the literature. Since both problems are NP-hard [9], many heuristic solution methods have been developed and evaluated under various assumptions and network settings [10], [11].

In this work, we are interested in obtaining optimal solutions to the static RWA problem. Several mixed integer linear program (MILP) formulations have been proposed in the literature for both the minRWA and maxRWA problems. In general, most conventional formulations can be categorized as either *link-based* (e.g., [12]) or *path-based* (e.g., [13]). Both link- and path-based formulations share the common drawback of being highly symmetrical with respect to wavelength permutations. Moreover, the problem size increases rapidly with the number of wavelengths, hence these formulations do not scale well to network environments that can be realized with current technology, which supports 120 or more wavelengths per link.

An alternative formulation was developed in [13] to capitalize on the fact that the wavelength assignment problem is equivalent to the graph multi-coloring problem. This formulation is based on maximal independent sets (MIS) and is such that the problem size is independent of the number of wavelengths. However, the number of maximal independent sets grows exponentially with the size n of the graph to be colored. For a general graph, the upper bound on the number of maximal independent sets is $3^{(n/3)}$. Note that, in the RWA problem formulation, the size of the graph is equal to the number of paths in the original network, which poses severe scalability challenges. Consequently, rather than solving the MIS formulation directly, the authors of [13] used its LP relaxation to obtain lower bounds. Even in ring networks, the number of paths grows exponentially with the number of nodes, limiting the application of the MIS formulation to small

This work was supported by a grant by the Scientific and Technological Research Council of Turkey (TUBITAK) and the Secure Open Systems Initiative (SOSI) at North Carolina State University.

networks.

To overcome this limitation, column generation techniques may be used. Column generation, first proposed in the context of graph coloring in [14], is an iterative technique which formulates the problem with a subset of MISs and adds any necessary additional variables on the fly by solving a second simpler LP. This techniques has also been applied to solve the RWA problem in [15], [16]. Although the column generation method does yield smaller problem sizes for each iteration, it nevertheless requires the computation of all the MISs and also involves solving an LP which includes all the variables. Consequently, it may not scale to realistic network sizes to be practical for network operators.

In this paper, we consider the static RWA problem in WDM ring networks. Although there is some evidence that network operators may transition to mesh networks, vast parts of the current optical network infrastructure are based on SONET/SDH rings. Furthermore, any such transition to mesh networks is likely to be slow and take place over many years. Therefore, optimal network design techniques for WDM rings are likely to be important for the foreseeable future.

Starting with the MIS formulation, we develop a decomposition approach to obtain an equivalent formulation with a much smaller number of variables. Our approach consists of partitioning the path set and representing the maximal independent sets in the original network using the independent sets calculated in each of these partitions. The result is a suite of formulations that trades off the number of variables with the number of constraints and, as a result, achieve a much better scalability in terms of network size. We present numerical results to demonstrate that our new formulation achieves more than two orders of magnitude reduction in running time compared to the link, path, or original MIS formulation. Specifically, we show that ring networks of least 16 nodes (the maximum size of a SONET/SDH ring) can be solved in just a few seconds. Therefore, our new approach has several unique practical benefits for network designers and operators, including: (1) the ability to solve the RWA problem optimally for any existing WDM ring network, and for any number of wavelengths; (2) the ability to perform extensive "what-if" analysis to evaluate the sensitivity of the optimal solution to uncertainties in forecast traffic demands; and (3) the potential to speed-up the solution of other hard network design problems for which RWA is a subproblem. While it may not be possible to obtain optimal solutions to all hard network design problems that include RWA as a subproblem, the capability of solving larger instances to optimality makes it possible to evaluate the performance of heuristics and develop more efficient ones.

The rest of the paper is organized as follows. In the next section, we introduce the network model and notation, and, for the sake of completeness, we present the earlier link, path, and MIS formulations of the minRWA problem. In Section III, we describe our new formulation based on decomposition of the maximal independent set. We present numerical results in Section IV, and we conclude in Section V.

II. Notation and Existing RWA Formulations

The physical topology of an optical network can be represented as a graph $G = (\mathcal{N}, \mathcal{L})$, where $\mathcal{N}$ is the set of N network nodes and $\mathcal{L}$ is the set of L physical links connecting the nodes. We assume that each physical link is directed and consists of a single fiber supporting W wavelength channels. Nodes are connected with two links in opposite directions. The amount of traffic demand from node s to node d, in terms of the number of lightpaths (connections) to be set up, is represented as t_{sd} and $T = [t_{sd}]$ forms the overall network traffic matrix.

The set of all node pairs in the network is denoted as $\mathcal{Z}$, i.e., $\mathcal{Z} = \{(i,j) : i,j \in \mathcal{N}, i \neq j\}$ and $Z = |\mathcal{Z}|$. In a ring network, there are two possible paths between a node pair $(i,j) \in \mathcal{Z}$: one in the clockwise and the other in the counter-clockwise direction, represented as $p_{ij,0}$ and $p_{ij,1}$, respectively. The set of all paths $\mathcal{P}$ is the union of the set of clockwise paths (denoted by $\mathcal{P}^0$) and the set of counter-clockwise paths (denoted by $\mathcal{P}^1$), where $\mathcal{P}^k = \{p_{ij,k}\}$ for $k = 0, 1$, and $P = |\mathcal{P}|$.

Using the above notation, the minRWA problem can be defined as determining the minimum number of wavelengths to satisfy all the demands in T, subject to the constraint that no two lightpaths sharing a common link use the same wavelength. On the other hand, the the maxRWA problem can be defined as maximizing the number of satisfied demands for a given number of wavelengths, subject to the same constraint. In the following subsections, we present link, path, and MIS formulations of the minRWA problem, using consistent notation. Due to space constraints, we omit the formulations for the maxRWA problem; however, these can be derived from the formulations presented here by appropriately adapting the objective function and some of the constraints.

A. Link Formulation

Denoting the set of links outgoing from (respectively, incoming to) node n as $\mathcal{L}_n^+$ (respectively, $\mathcal{L}_n^-$), the minRWA formulation can be stated as:

$$\min V$$

subject to

$$\sum_{l \in \mathcal{L}_n^+} c_{ij}^{w,l} - \sum_{l \in \mathcal{L}_n^-} c_{ij}^{w,l} = \begin{cases} 0 & n \neq i,j \\ t_{ij} & n = i \\ -t_{ij} & n = j \end{cases} \quad \begin{array}{l} \forall n \in \mathcal{N}, \\ (i,j) \in \mathcal{Z}, w \end{array} \tag{1}$$

$$\sum_{(i,j) \in \mathcal{Z}} c_{ij}^{w,l} \leq 1 \quad \forall l \in \mathcal{L}, \forall w \tag{2}$$

$$\sum_{(i,j) \in \mathcal{Z}} \sum_{l \in \mathcal{L}} c_{ij}^{w,l} \leq u^w Z L \quad \forall w \tag{3}$$

$$V \geq w u^w \quad \forall w \tag{4}$$

where $c_{ij}^{l,w} = 1$, if there exists a lightpath from node i to node j that uses wavelength w on link l, and is 0 otherwise. u^w is a binary variable which indicates whether wavelength w is used and V is the number of wavelengths used. Expressions (1) are the multi-commodity flow equations corresponding to the routing subproblem and expression (2) is the wavelength constraint. Constraints (3) ensure that u^w is 1 whenever wavelength w is used by any lightpath on any node and expression (4) sets V to the index of the largest wavelength used. In the actual implementation, we use separate constraints for the incoming and outgoing lightpaths at source and destination nodes in (1) to improve efficiency.

B. Path Formulation

For ring networks, there are only two possible paths between each node pair. Hence, the routing subproblem reduces to selecting either the clockwise or the counter-clockwise path for each lightpath between a node pair, which results in significant reduction in problem size compared to arbitrary network topologies. The path formulation for the minRWA problem is given as:

$$\min V$$

subject to

$$\sum_{k=0,1} \sum_{w} c^w_{ij,k} = t_{ij} \quad \forall (i,j) \in \mathcal{Z} \tag{5}$$

$$\sum_{(i,j)\in\mathcal{Z}} \sum_{k=0,1} c^w_{ij,k} X^l_{ij,k} \leq 1 \quad \forall l \in \mathcal{L}, \forall w \tag{6}$$

$$\sum_{(i,j)\in\mathcal{Z}} \sum_{k=0,1} c^w_{ij,k} \leq u^w P \quad \forall w \tag{7}$$

$$V \geq w u^w \quad \forall w \tag{8}$$

where $c^w_{ij,k}$ is the binary decision variable indicating whether there exists a lightpath on path $p_{ij,k}$ which uses wavelength w. The variable $X^l_{ij,k} = 1$, if $p_{ij,k}$ is uses link l, and is 0 otherwise. Expression (5) ensures that all demands are satisfied, while expression (6) is the wavelength constraint.

C. MIS Formulation

The wavelength assignment problem can be transformed into a graph multi-coloring problem by defining a new graph G_p where each node corresponds to a path in G and two nodes are connected to each other in G_p if the corresponding paths in G share a common link. The problem is then equivalent to assigning separate colors to a node in G_p for each lightpath established over the corresponding path in G, such that the two adjacent nodes are not assigned the same color. Thus, a set of paths in G can be assigned the same wavelength if the corresponding nodes in G_p form an independent set.

We denote the number of lightpaths on path $p_{ij,k}$ as $b_{ij,k}$, and let v_m be the number of wavelengths assigned to the independent set m. Let $\mathcal{M}$ denote the set of all maximal independent sets in G_p, which can be calculated efficiently using the Bron-Kerbosch algorithm [17]. Also, let $Y^m_{ij,k}$ be the path-path set incidence function defined as

$$Y^m_{ij,k} = \begin{cases} 1, & \text{if path set } m \text{ contains path } p_{ij,k}, \\ 0, & \text{otherwise.} \end{cases} \tag{9}$$

The ILP formulation can now be written as

$$\min V$$

subject to

$$\sum_{k=0,1} b_{ij,k} = t_{ij} \qquad \forall (i,j) \in \mathcal{Z} \tag{10}$$

$$b_{ij,k} \leq \sum_{m\in\mathcal{M}} v_m Y^m_{ij,k} \qquad \forall (i,j) \in \mathcal{Z}, k = 0,1 \tag{11}$$

$$\sum_{m\in\mathcal{M}} v_m \leq V \tag{12}$$

The first set of constraints ensures that the traffic demand between each node pair is satisfied by using lightpaths over the clockwise and counter-clockwise paths. Since, the number of wavelengths assigned to a path is the sum of the number of wavelengths assigned to maximal independent sets which include that path, the second set of constraints ensures that each path is assigned a sufficient number of wavelengths.

The MIS formulation has the clear advantage of being independent of the number W of wavelengths, whereas the sizes of the path- and link-based formulations increase with W. Moreover, the link and path formulations have a symmetry problem. Specifically, given a feasible solution, different solutions with the same objective value can be obtained by simply changing the order of wavelengths. This means that there are $W!$ different optimal solutions to the problem; since the ILP solver has to evaluate all of these solutions, the running time can be unnecessarily long. On the other hand, the number of MISs in G_p increases exponentially with the number of paths, which in turn increases quadratically with the number of nodes in G. Therefore, the number of variables, v_m, grows rapidly with the size of the network, limiting the applicability of the MIS formulation to small networks.

In the following section, we develop methods to obtain formulations *equivalent* to (10)-(12) using a smaller number of variables, so that larger network instances can be solved without sacrificing the benefits of the MIS-based formulation.

III. Maximal Independent Set Decomposition (MISD) and RWA Formulations based on MISD

As we discussed in Section II-C, the limiting factor for the MIS-based RWA formulation is the exponential increase in the number of MISs with the number of nodes (vertices) of G_p. In this section, we propose a new approach to decrease the number of independent sets. The method is based on identifying two nearly equal and preferably large independent subgraphs in the path graph, G_p, and dividing G_p into three components, two of which are the identified independent subgraphs and the third subgraph includes the rest of the nodes in G_p. As will be discussed shortly, this graph partitioning enables an efficient decomposition of MISs in G_p. Importantly, such graphs can be easily identified in ring networks by using the path-link incidence information. Then, the MISs in G_p are represented as combinations of independent sets in each subgraph. The result is a very beneficial trade-off between the number of variables and constraints in the RWA formulation.

A. Maximal Independent Set Decomposition with 2 Independent Path Sets (MISD-2)

Realizing that the clockwise paths do not intersect with counter-clockwise paths, G_p can be divided into two disconnected components, G_p^0 and G_p^1, corresponding to the sets $\mathcal{P}^0$ and $\mathcal{P}^1$, respectively. The MISs in G_p^0 (respectively, G_p^1) are denoted as $\mathcal{M}^0$ (respectively, $\mathcal{M}^1$). Also, we define v_m^k as the number of wavelengths assigned to the MIS $m \in \mathcal{M}^k$ for $k = 0, 1$. Then (11) and (12) in the basic MIS formulation are replaced with:

$$b_{ij,k} \leq \sum_{m \in \mathcal{M}^k} v_m^k X_{ij,k}^m \qquad \forall (i,j) \in \mathcal{Z}, k = 0,1 \quad (13)$$

$$\sum_{m \in \mathcal{M}^k} v_m^k \leq V \qquad k = 0,1 \quad (14)$$

Note that, for each $m_i \in \mathcal{M}^0$ and $m_j \in \mathcal{M}^1$, $m_i \cup m_j$ gives an MIS for G_p.

B. Maximal Independent Set Decomposition with 4 Independent Path Sets (MISD-4)

In order to further decrease the number of variables in the formulation, G_p^k $(k = 0, 1)$ is divided further into three partitions: $G_p^{k,core}$, $G_p^{k,0}$, and $G_p^{k,1}$. The partitions are selected such that there are no links between the nodes in $G_p^{k,0}$ and the nodes in $G_p^{k,1}$. The remaining nodes are collected in the set $G_p^{k,core}$. This operation is equivalent to partitioning the path set $\mathcal{P}^k$ into three subsets, where none of the paths in $\mathcal{P}^{k,0}$ intersect with any of the paths in $\mathcal{P}^{k,1}$. Also, $\mathcal{P}^{k,core}$ includes the remaining paths in $\mathcal{P}^k$, which may intersect with the paths in $\mathcal{P}^{k,0}$ and/or $\mathcal{P}^{k,1}$.

For the ring network case, an appropriate partitioning can be obtained based on the links that each path uses. Assuming that nodes in the ring network are numbered from 1 to N in the clockwise direction, and denoting the clockwise (respectively, counter-clockwise) links as $\mathcal{L}^0$ (respectively, $\mathcal{L}^1$):

- $\mathcal{P}^{k,0} \subset \mathcal{P}^k$, is defined as the set of paths that use only links in $\mathcal{L}^k$ between nodes $\{1, \ldots, \lfloor N/2 \rfloor\}$.
- $\mathcal{P}^{k,1} \subset \mathcal{P}^k$, is defined as the set of paths that use only the rest of the links in $\mathcal{L}^k$.
- $\mathcal{P}^{k,core} \subset \mathcal{P}^k$, consists of the paths that use links from both parts.

This partitioning results in 4 independent path sets, namely $\mathcal{P}^{0,0}$, $\mathcal{P}^{0,1}$, $\mathcal{P}^{1,0}$ and $\mathcal{P}^{1,1}$.

For developing the formulation, it is necessary to introduce a new set definition, referred to as *core set*. Core sets for G_p^k are denoted as $\mathcal{Q}^k$ and defined as the sets of nodes in $G_p^{k,core}$ which are maximal subsets of any MIS in G_p^k. In other words, $\mathcal{Q}^k$ includes the intersection of any MIS in G_p^k with the node set $G_p^{k,core}$, as an element. Consequently, any MIS in G_p^k can be written as the union of a set in $\mathcal{Q}^k$ with some nodes in $G_p^{k,0}$ and/or $G_p^{k,1}$. The core sets are calculated using the following Algorithm 1. The running time complexity of the algorithm is $O(|\mathcal{Q}^k|N^2)$.

Algorithm 1 Calculation of core sets $\mathcal{Q}^k$.

```
Initialize Q^k = maximal independent sets in G_p^{k,core}.
for each core set q ∈ Q^k do
  for each node p ∈ q do
    if p has a link to a node r ∈ G_p^{k,0} ∪ G_p^{k,1}, and none of
    the nodes in q \ {p} have a link to that node then
      Append the set q \ {p} to Q^k.
    end if
  end for
end for
Add ∅ to Q^k.
```

Finally, for each core set $q \in \mathcal{Q}^k$, the maximal sets of nodes in $G_p^{k,k'}$ which are independent from each other and the nodes in q $(\mathcal{M}_q^{k,k'})$ are calculated for $k, k' = 0, 1$.

With these definitions, for each $q \in \mathcal{Q}^k$ and $m_i \in \mathcal{M}_q^{k,0}$ and $m_j \in \mathcal{M}_q^{k,1}$, $m_i \cup q \cup m_j$ corresponds to an MIS in G_p^k.

The MISD-4 formulation can now be obtained by replacing (11) and (12) in the basic MIS formulation with the following equations:

$$b_{ij,k} \leq \sum_{q \in \mathcal{Q}^k} v_q^{k,core} X_{ij,k}^q \qquad \forall p_{ij,k} \in \mathcal{P}^{k,core}, k = 0,1 \quad (15)$$

$$b_{ij,k} \leq \sum_{q \in \mathcal{Q}^k} \sum_{m \in \mathcal{M}_q^{k,k'}} v_{q,m}^{k,k'} X_{ij,k}^m \qquad \forall p_{ij,k} \in \mathcal{P}^{k,k'}, \; k, k' = 0,1 \quad (16)$$

$$\sum_{q \in \mathcal{Q}^k} v_q^{k,core} \leq V \qquad k = 0,1 \quad (17)$$

$$\sum_{m \in \mathcal{M}_q^{k,k'}} v_{q,m}^{k,k'} = v_q^{k,core} \qquad \forall q \in Q^k, k' = 0,1 \quad (18)$$

In this formulation, $v_q^{k,core}$ is the number of wavelengths assigned to the core set $q \in \mathcal{Q}^k$ and $v_{q,m}^{k,k'}$ denotes the number of wavelengths assigned to the set $m \in \mathcal{M}_q^{k,k'}$. Expressions (15) and (16) ensure that each set is assigned a sufficient number of wavelengths that the number of wavelengths on each path is greater than or equal to the number of lightpaths on that path. Expression (17) sets V to the number of wavelengths used, while constraints (18) ensure consistency between wavelength assignment in different path partitions.

This decomposition approach can be further extended to develop formulations with 8 (MISD-8), 16 (MISD-16) or more independent path sets. Due to page constraints, the exact formulations are omitted.

C. Comparison of the MIS and MISD-x Formulations

In Fig. 1 we plot the number of independent sets in the basic MIS formulation, as well as the MISD-2, MISD-4, and MISD-8 formulations, using a logarithmic y-axis, against the number N of ring nodes. We observe that the number of independent sets in MISD-2 is just the square root of the corresponding number in the basic MIS formulation. This is due to the fact that the path graph for ring network is composed of two

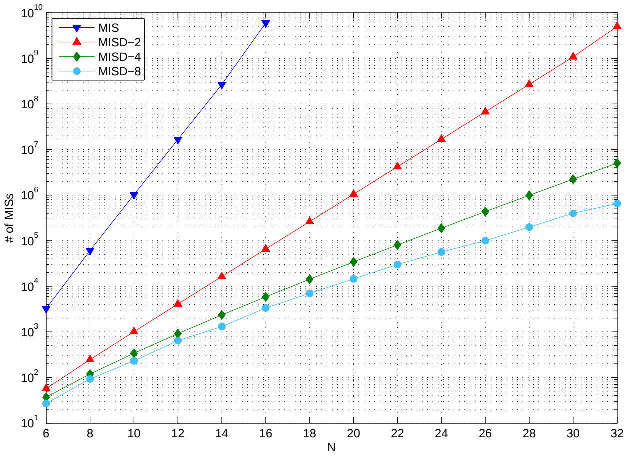

Fig. 1. Comparison of formulations in terms of MIS decision variables

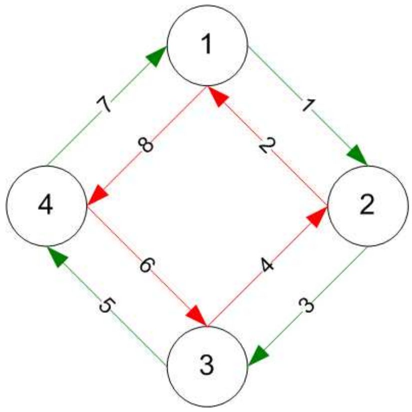

Fig. 2. The 4-node ring network for the example in Section III-D

TABLE I
THE SET OF PATHS BETWEEN EACH NODE PAIR IN THE 4-NODE RING NETWORK OF FIG. 2

path #	i	j	k	links	path #	i	j	k	links
1	1	2	0	1	13	3	1	0	5-7
2	1	2	1	8-6-4	14	3	1	1	4-2
3	1	3	0	1-3	15	3	2	0	5-7-1
4	1	3	1	8-6	16	3	2	1	4
5	1	4	0	1-3-5	17	3	4	0	5
6	1	4	1	8	18	3	4	1	4-2-8
7	2	1	0	3-5-7	19	4	1	0	7
8	2	1	1	2	20	4	1	1	6-4-2
9	2	3	0	3	21	4	2	0	7-1
10	2	3	1	2-8-6	22	4	2	1	6-4
11	2	4	0	3-5	23	4	3	0	7-1-3
12	2	4	1	2-8	24	4	3	1	6

disconnected subgraphs. We also note that the MISD-4 and MISD-8 formulations achieve a further significant reduction in the number of MISs. For instance, on a 16-node ring network, the number of MISs in MISD-4 is nearly an order of magnitude smaller than in MISD-2, while for a 32-node ring, the corresponding reduction in maximal independent sets is nearly three orders of magnitude.

This decrease in MIS size comes at the expense of additional constraints (i.e., those corresponding to expression (18)), the number of which is equal to the total number of core sets. However, the number of additional constraints is low relative to to the great reduction in the number of independent sets. As an example, for a 16-node ring, the number of core sets in the MISD-4 formulation is just 953. As a result, by adding a small number of constraints, MISD-4 successfully eliminates a large number of variables in the MILP formulation.

D. Illustrative Example

To better clarify the operation of the MISD algorithms, in this section we present a simple illustration using the 4-node ring network depicted in Fig. 2. Each link of the ring is associated with an ID shown in the figure. The set of all paths are listed in Table I.

MIS Formulation. The basic algorithm calculates the set of MISs, $\mathcal{M}$, using the whole set of paths, $\mathcal{P}$, without partitioning. The number of MISs is found to be 121.

MISD-2 Formulation. The MISD-2 algorithm partitions the sets into two subsets. The set of clockwise paths is

$$\mathcal{P}^0 = \{1,3,5,7,9,11,13,15,17,19,21,23\},$$

and the set of counter-clockwise paths is

$$\mathcal{P}^1 = \{2,4,6,8,10,12,14,16,18,20,22,24\}.$$

Then, the MISs in $\mathcal{P}^0$ and $\mathcal{P}^1$ are calculated as:
$\mathcal{M}^0 = \{\{1,9,17,19\},\{1,9,13\},\{1,7\},\{1,11,19\},\{3,17,19\},$ $\{3,13\},\{5,19\},\{15,9\},\{21,9,17\},\{21,11\},\{23,17\}\}$,
$\mathcal{M}^1 = \{\{8,16,24,6\},\{8,16,4\},\{8,2\},\{8,22,6\},\{14,24,6\},$ $\{14,4\},\{20,6\},\{10,16\},\{12,16,24\},\{12,22\},\{18,24\}\}$, respectively.

The set $\mathcal{M}$ is equal to the cross product of sets in $\mathcal{M}^0$ and $\mathcal{M}^1$ (each of size 11 , and is equal to the set $\mathcal{M}$ (of size 121) for the original MIS formulation above.

MISD-4 Formulation. The MISD-4 algorithm partitions the clockwise paths into 2 independent sets and a core set as follows:

- $\mathcal{P}^{0,0} = \{1,3,9\}$: the set of paths that use only the first two clockwise links, links 1,3.
- $\mathcal{P}^{0,1} = \{13,17,19\}$: the set of paths that use the other two clockwise links, links 5,7, only.
- $\mathcal{P}^{0,core} = \{5,7,11,15,21,23\}$: the remaining set of paths that use any of the four clockwise links, links 1,3,5, and 7.

Then, the set of core sets for the clockwise paths, $\mathcal{Q}^0$, is calculated as follows:

- Initialize $\mathcal{Q}^0$ to MIS set of $G_p^{0,core}$: $\mathcal{Q}^0 = \{\{11,21\},\{7\},\{5\},\{15\},\{23\}\}$
- For $q = \{11,21\}$, path 11 intersects with path 3, but path 21 does not intersect with path 13; hence, set $\{21\}$ is added to $\mathcal{Q}^0$.
- For $q = \{11,21\}$, path 21 intersects with path 19, but path 11 does not intersect with path 19; thus, set $\{11\}$ is

TABLE II
INDEPENDENT SETS IN $\mathcal{G}_p^{0,0}$ AND $\mathcal{G}_p^{0,1}$ CORRESPONDING TO EACH CORE SET $q \in \mathcal{Q}^0$

q	$\mathcal{M}_q^{0,0}$	$\mathcal{M}_q^{0,1}$
$\{11, 21\}$	$\{\{\}\}$	$\{\{\}\}$
$\{7\}$	$\{\{1\}\}$	$\{\{\}\}$
$\{5\}$	$\{\{\}\}$	$\{\{19\}\}$
$\{15\}$	$\{\{9\}\}$	$\{\{\}\}$
$\{23\}$	$\{\{\}\}$	$\{\{17\}\}$
$\{21\}$	$\{\{9\}\}$	$\{\{17\}\}$
$\{11\}$	$\{\{1\}\}$	$\{\{19\}\}$
$\{\}$	$\{\{1, 9\}, \{3\}\}$	$\{\{17, 19\}, \{13\}\}$

added to $\mathcal{Q}^0$.
- $\emptyset$ is added to $\mathcal{Q}^0$

As a result,

$$\mathcal{Q}^0 = \{\{11, 21\}, \{7\}, \{5\}, \{15\}, \{23\}, \{21\}, \{11\}, \{\}\}$$

Then, for each $q \in \mathcal{Q}^0$, $\mathcal{M}_q^{0,0}$ and $\mathcal{M}_q^{0,1}$ are found as given in Table II.

Comparing with the set $\mathcal{M}^0$ obtained above for MISD-2, we observe that, for each $q \in \mathcal{Q}^0$, $m_i \in \mathcal{M}_q^{0,0}$ and $m_j \in \mathcal{M}_q^{0,1}$, $m_i \cup q \cup m_j$ corresponds to an MIS in graph G_p^0 corresponding to set P^0. The sets $\mathcal{Q}^1$, $\mathcal{M}_q^{1,0}$, and $\mathcal{M}_q^{1,1}$ are similarly obtained for the counter-clockwise paths.

IV. NUMERICAL RESULTS

We now present numerical results to compare the efficiency of the link, path, MIS, MISD-2, MISD-4, and MISD-8 formulations of the RWA problem we presented in Sections II and III. To this end, we used the CPLEX 11 optimization software to solve the corresponding formulations of identical problem instances on a cluster of compute nodes with dual Woodcrest Xeon processors running at 2.33GHz with 1333 MHz memory bus, 4GB of memory and 4MB L2 cache.

In our comparisons, we used a large set of random problem instances that were generated by varying the number N of nodes in the ring network ($N = 6, 7, \ldots, 24$), the number W of wavelengths per link ($W = 10, 20, \ldots, 160$), and the traffic demands t_{sd} (in lightpaths) between the various source-destination pairs (s, d) in the network. We also imposed a time limit of 2 CPU hours for CPLEX to find a solution for a given formulation and problem instance; if it failed to do so within the 2-hour limit, we terminated the execution run and report this fact in the figures shown in this section.

Fig. 3 compares the various formulations of minRWA in terms of the CPU time (in log scale) it takes for CPLEX to find an optimal solution, against the size N of the ring network; similar results for maxRWA are omitted. Each data point in the figure represents the average of 30 random instances generated by drawing traffic demands (in lightpaths) uniformly at random in the interval $[0, 5]$. The data points in the light gray area of the figure labeled “tLim” correspond to instances that could not be solved within the 2-hour time limit we mentioned above. On the other hand, the data points in the top dark gray area of the figure labeled “Mem” correspond to instances for

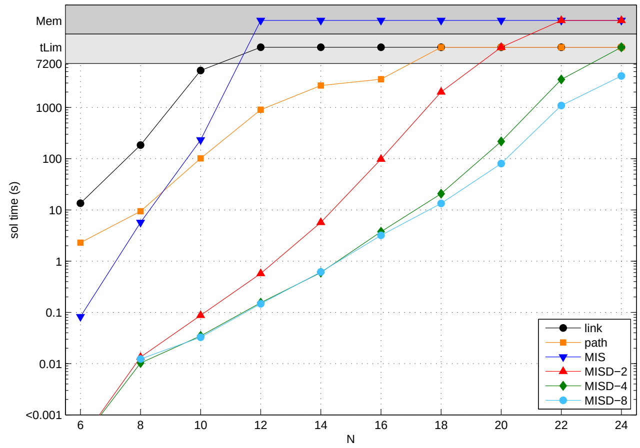

Fig. 3. Solution times of minRWA formulations as a function of N, $W = 120$

which the formulation could not fit in the available memory for CPLEX to run.

The link formulation fails to solve instances with $N > 10$ nodes within the time limit. The path formulation is more efficient: CPLEX is able to find the optimal solution for $N \leq 16$, but the running time exceeds the 2-hour limit for all instances with $N > 16$. MIS runs faster than the path and link formulations up to 8 and 10 nodes, respectively. However, the formulation size gets too large for CPLEX to solve beyond 10 nodes. The new MISD formulations perform much better, with running times below 1 sec up to 14-15 nodes (for MISD-4 and MISD-8), several orders of magnitude less than the other three. Beyond 12 nodes, MISD-2 performs noticeably worse than MISD-4 and MISD-8, and beyond 20 nodes its size becomes too large to fit in memory; similarly, MISD-8 starts outperforming MISD-4 for networks with $N \geq 18$ nodes. MISD-8 is able to obtain the optimal solution for 24 nodes, 8 nodes more than the path formulation in about the same amount of time.

From a practical perspective, MISD-4 and MISD-8 make it possible to solve RWA optimally for a maximum-size (16-node) SONET ring in only a few (i.e., 3-4) seconds. Such an instance can only be tackled by the path formulation, but takes CPLEX almost two hours, on average, to find the optimal solution. Consequently, MISD-4 and MISD-8 allow network designers and operators to perform extensive “what-if” analysis by investigating large numbers of scenarios regarding forecast demands, cost and price structures, etc; such analysis would either not be possible previously, or would require vast amounts of computational resources and time.

Fig. 4 presents another set of experiments we performed to determine the maximum number of nodes in a problem instance that can be solved by each formulation for different values of the number W of wavelengths within 3000 sec. The problem instances were generated in the same manner as those shown in Fig. 3. The dark gray area in the figure

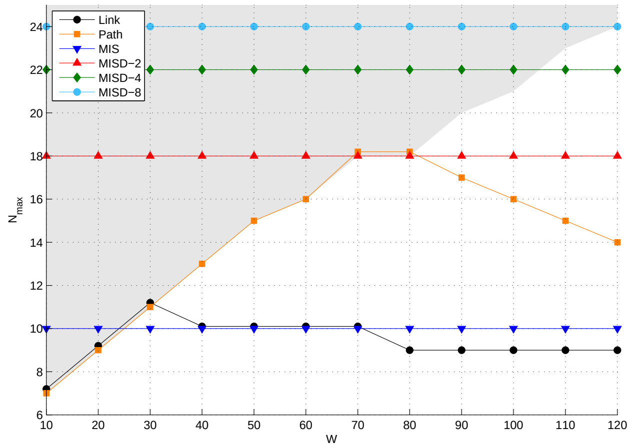

Fig. 4. Size of the largest network size N_{max} that can be solved with each minRWA formulation for a given number W of wavelengths, within 3000 seconds of CPU time

denotes instances that are infeasible. Note that, since the MIS and MISD-z formulations are independent of W, the corresponding curves are straight lines parallel to the x-axis; also, these formulations find optimal solutions even when the problem is infeasible for the indicated number of wavelengths (of course, the optimal solution in this case is a value of W higher than the indicated one). As we can see, among the MIS-based formulations, MISD-8 has the best scalability (it can find solutions for $N = 24$ nodes), followed by MISD-4 and MISD-2 (the latter can find solutions for $N = 16$ nodes, i.e., a maximum size SONET ring), while MIS can only solve instances up to $N = 10$ nodes due to excessive memory requirements. Within the 3000 sec limit, the link formulation can obtain solutions for up to $N = 10-11$ nodes for moderate number of wavelengths, but as W increases, it is limited to very small networks. Finally, the path formulation performs better than the link formulation, but is also severely restricted as W increases to the limits of current technology.

In summary, the results we have presented in this section suggest that the MISD formulations can provide significant time savings in solving minRWA and maxRWA problem instances on realistic size networks, and they are able to increase the network sizes that can be solved to optimality.

V. Concluding Remarks

RWA is one of the most important and classical problems arising in the design of WDM networks, and it has been extensively studied. However, with the increasing number of wavelengths supported by optical transmission technology, existing formulations face significant scalability challenges. We have developed an independent set formulation based on graph partitioning that has the advantage of being independent of the number of wavelengths, and has better scalability than existing MIS formulations. We have demonstrated that the new approach enables the solution of problems several orders of magnitude faster than the conventional methods and is able to solve larger network instances to optimality. We are currently working to extend this work in two directions: (1) develop efficient MIS decomposition techniques for networks of general topology, and (2) investigate the impact of these more efficient RWA formulations on the complexity of other important network design problems, including traffic grooming, which include RWA as a subproblem.

References

[1] R. Dutta, A. E. Kamal, and G. N. Rouskas, Eds., *Traffic Grooming in Optical Networks: Foundations, Techniques, and Frontiers*. Springer, 2008.

[2] R. Dutta and G. Rouskas, "Traffic grooming in WDM networks: Past and future," *IEEE Netw.*, vol. 16, no. 6, pp. 46–56, Nov./Dec. 2002.

[3] K. Zhu and B. Mukherjee, "Traffic grooming in optical WDM mesh networks," *IEEE J. Sel. Areas Commun.*, vol. 20, no. 1, pp. 122–133, Jan. 2002.

[4] S. Ramamurthy, L. Sahasrabuddhe, and B. Mukherjee, "Survivable WDM mesh networks," *Journal of Lightwave Technology*, vol. 21, no. 4, pp. 870–883, April 2003.

[5] S. Ramamurthy and B. Mukherjee, "Survivable WDM mesh networks, part I – protection," in *Proceedings of INFOCOM '99*, March 1999, pp. 744–751.

[6] J. Kuri, N. Puech, M. Gagnaire, E. Dotaro, and R. Douville, "A review of routing and wavelength assignment of scheduled lightpath demands," *IEEE Journal Selected Areas in Communications*, vol. 21, no. 8, pp. 1231–1240, October 2003.

[7] W. Su and G. Sasaki, "Scheduling periodic transfers with flexibility," in *Proceedings of 41st Allerton Conference*, October 2003.

[8] B. Jaumard, C. Meyer, B. Thiongane, and X. Yu, "ILP formulations and optimal solutions for the RWA problem," in *Proc. IEEE GLOBECOM'04*, vol. 3, Nov.29–Dec.3 2004, pp. 1918–1924.

[9] I. Chlamtac, A. Ganz, and G. Karmi, "Lightpath communications: An approach to high bandwidth optical WANS," *IEEE Transactions on Communications*, vol. 40, no. 7, pp. 1171–1182, July 1992.

[10] R. Dutta and G. Rouskas, "A survey of virtual topology design algorithms for wavelength routed optical networks," *Optical Networks Magazine*, vol. 1, no. 1, pp. 73–89, Jan. 2000.

[11] H. Zang, J. Jue, and B. Mukherjee, "A review of routing and wavelength assignment approaches for wavelength-routed optical WDM networks," *Optical Networks Magazine*, vol. 1, no. 1, pp. 47–60, Jan. 2000.

[12] R. M. Krishnaswamy and K. N. Sivarajan, "Algorithms for routing and wavelength assignment based on solutions of LP-relaxations," *IEEE Commun. Lett.*, vol. 5, no. 10, pp. 435–437, Oct. 2001.

[13] R. Ramaswami and K. Sivarajan, "Routing and wavelength assignment in all-optical networks," *IEEE/ACM Trans. Netw.*, vol. 3, no. 5, pp. 489–500, Oct. 1995.

[14] A. Mehrotra and M. Trick, "A column generation approach for graph coloring," *INFORMS Journal on Computing*, vol. 8, no. 4, pp. 344–354, Fall 1996.

[15] T. Lee, K. Lee, and S. Park, "Optimal routing and wavelength assignment in WDM ring networks," *IEEE J. Sel. Areas Commun.*, vol. 18, no. 10, pp. 2146–2154, Oct. 2000.

[16] B. Jaumard, C. Meyer, and B. Thiongane, "On column generation formulations for the RWA problem," *Discrete Applied Mathematics*, vol. 157, no. 6, pp. 1291–1308, Mar. 2009.

[17] C. Bron and J. Kerbosch, "Algorithm 457: Finding all cliques of an undirected graph," *Communications of the ACM*, vol. 16, no. 9, pp. 575–577, Sept. 1973.