Computing Blocking Probabilities in Multiclass Wavelength-Routing Networks With Multicast Calls

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***Abstract*—We present an approximate analytical method to compute efficiently the call-blocking probabilities in wavelength-routing networks with multiple classes of both unicast and multicast calls. Our approach involves the following steps. We start with an approximate solution to a linear single-class unicast network which we developed earlier. Next, all classes of calls on a particular route are aggregated to give an equivalent single-class model. We then extend the path decomposition algorithms that we have developed for single-class networks to handle mesh networks with multiple classes of calls. We show how to use these path decomposition algorithms to decompose large networks with multicast paths into smaller subsystems with only linear paths, which, in turn, are solved by the product-form approximation algorithm. We also consider a state-dependent Poisson arrival process for multicast calls which is more accurate in capturing the behavior of these calls.**

***Index Terms*—Blocking probabilities, multicast, multiclass networks, wavelength-division multiplexing (WDM), wavelength routing.**

I. INTRODUCTION

OPTICAL networks with wavelength-division multiplexing (WDM) have the potential of delivering an aggregate throughput in the order of Terabits per second, and they appear as a viable approach to satisfying the ever-growing demand for more bandwidth per user on a sustained, long-term basis [3]. An optical network architecture that appears promising for backbone networks is the one based on the concept of wavelength routing. The architecture consists of wavelength routers interconnected by fiber links. A wavelength router is capable of switching a light signal at a given wavelength from any input port to any output port. By appropriately configuring the routers, all-optical paths (lightpaths) may be established in the network. Lightpaths represent direct optical connections without any intermediate electronics. Because of the long propagation delays, and the time required to configure the routers, wavelength-routing networks are expected to operate in circuit-switched mode.

A significant amount of research effort has been devoted to addressing the set of issues that arise in the design of wavelength-routing networks [3]. One problem that has attracted considerable attention is that of computing call-blocking probabilities in such networks. Several variations of this problem have been studied, mainly differing in the underlying assumptions regarding the availability, type, and location of converters, the routing algorithm, the wavelength allocation policy, the traffic type, and the arrival process. A major difficulty in analyzing wavelength-routing networks is the tight coupling between routing and wavelength allocation, which is not present in conventional circuit-switched networks and which requires the development of new analytical techniques.

The problem of computing call-blocking probabilities under static or fixed alternate routing with random wavelength allocation and with or without converters has been considered in several studies, including [1], [2], [5], and [10]. Other wavelength allocation schemes, as well as dynamic routing, are harder to analyze and have been considered in [6], [8], and [13]. All of the above studies assume that either all or none of the wavelength routers have full wavelength conversion capabilities. The case of *sparse* conversion, whereby only a subset of the set of nodes are equipped with converters was studied in [10], while *limited* conversion was considered in [11], where it is assumed that all nodes are equipped with devices which can convert each wavelength to only a subset of the supported wavelengths. Finally, the problem of routing and wavelength assignment when the network carries multicast calls is addressed in [9], while an analysis of call-blocking probabilities in multifiber networks appeared in [7].

Most of the approximate analytical techniques developed for computing blocking probabilities in wavelength-routing networks and discussed above amount to the well-known *link decomposition* approach [4]. Further, the development of some of the techniques is based on additional assumptions, for instance, that statistics of link loads are mutually independent, that link blocking events are independent, or that wavelength use on each link is characterized by a fixed probability independent of other wavelengths and links. Link decomposition has been extensively used in conventional circuit-switched networks where there is no requirement for the *same* wavelength to be used on successive links of the path taken by a call. The accuracy of these underlying approximations also depends on the traffic load, the network topology, and the routing and wavelength allocation schemes employed. While link decomposition techniques make it possible to study the qualitative behavior of wavelength-routing networks, more accurate analytical tools are needed to evaluate the performance of these networks efficiently, as well as to tackle complex network design problems, such as selecting the nodes where to employ converters.

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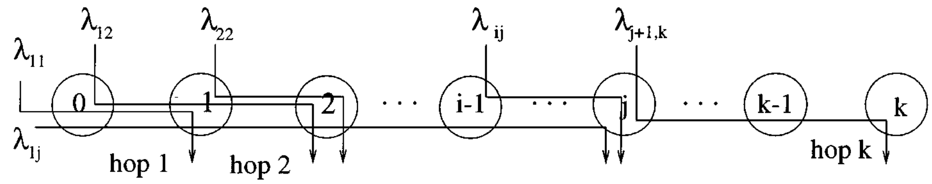

Fig. 1. A k-hop path.

We have developed an iterative *path decomposition* algorithm [15], [14] for analyzing wavelength-routing networks with sparse conversion. A network with point-to-point calls is decomposed into a number of path subsystems, each of which is analyzed in isolation using an approximate algorithm. The individual solutions are appropriately combined to form a solution for the overall network, and the process repeats until the blocking probabilities converge. Our approach accounts for the correlation of both link loads and link blocking events, giving accurate results for a wide range of loads and network topologies. In this paper we extend the decomposition algorithm in [14] to evaluate the blocking performance of optical networks with multiple classes of calls and multicast traffic. While previous studies have mostly focused on single-class wavelength-routing networks, it is expected that future networks will be utilized by a wide range of applications with varying characteristics in terms of their arrival rates and call holding times. To the best of our knowledge, this is the first time that multiclass wavelength-routing networks with multicast calls have been analyzed.

In Section II, we describe the network model. In Section III, we consider multiclass networks with unicast traffic, and in Section IV we extend the analysis to networks with multicast traffic. We present numerical results in Section V, and we conclude the paper in Section VI.

II. The Multiclass Wavelength-Routing Network

We consider a wavelength-routing network with an arbitrary topology in which each link supports exactly W wavelengths. Unicast call requests arrive according to a Poisson process with a rate that depends on the source-destination pair. In addition, there are multicast calls between a source and a set of destination nodes. The arrival of multicast calls is modeled by a state-dependent Poisson process. There are R different classes of calls, and an arriving multicast or unicast call belongs to one of theses classes. Call-holding times are assumed to be exponentially distributed, with a mean that depends on the class of the call, as well as on the source and destination nodes involved.

Some of the nodes in the network may employ wavelength converters. These nodes can switch an incoming wavelength to an arbitrary outgoing wavelength. If no converters are employed in the path between the source and the destination, a unicast call can only be established if the *same* wavelength is free on all the links used by the call; otherwise, the call is blocked. Similarly, a multicast call can only be established in a network without converters if the same wavelength is free on all links of the multicast tree connecting the source to the destinations. If a call can be accommodated, it is randomly assigned one of the wavelengths that are available on the links used by the call. In a network with converters, a wavelength is randomly assigned within each segment of a path whose starting and ending nodes are equipped with converters.

Since our model is an extension of a model developed for a single path, we introduce some relevant notation. A k-hop path (see Fig. 1) consists of $k+1$ nodes. Nodes $(i-1)$ and $i, 1 \le i \le k$ are said to be connected by link (hop) i. Calls originating at node $i-1$ and terminating at node j use hops i through $j, j \ge i \ge 1$, which we shall denote by the pair (i, j). Calls between these two nodes may belong to one of R classes, and these calls are said to use *route* (i, j). We also define the following parameters for a path with no converters.

- $\lambda_{ij}^{(r)}, j \ge i, 1 \le r \le R$ is the Poisson arrival rate of calls of class r that originate at node $(i-1)$ and terminate at node j.
- $1/\mu_{ij}^{(r)}$ is the mean of the exponentially distributed service time of calls of class r that originate at node $(i-1)$ and terminate at node j. We also let $\rho_{ij}^{(r)} = \lambda_{ij}^{(r)}/\mu_{ij}^{(r)}$.
- $N_{ij}^{(r)}(t)$ is the number of active calls at time t that use route (i, j) belonging to class r.
- $F_{ij}(t)$ is the number of wavelengths that are free on all hops of segment (i, j) at time t. A call that arrives at time t and uses route (i, j) is blocked if $F_{ij}(t) = 0$.

III. Multiclass Networks With Unicast Traffic

A. The Single-Class Single-Path Network

For completeness, in this section we review our previous results [14] for a single path of a network without converters; if there are converters in some of the nodes in the path the state description is slightly simplified, but everything else remains valid, as shown in [14]. Consider the k-hop path shown in Fig. 1. Let the state of this system at time t be described by the k^2-dimensional process

$$\underline{X}_k(t) = (N_{11}(t), N_{12}(t), \ldots, N_{kk}(t), F_{12}(t), \ldots, F_{1k}(t), F_{23}(t), \ldots, F_{(k-1)k}(t)). \quad (1)$$

Process $\underline{X}_k(t)$ is not time-reversible (see[14]), and this result is true in general, when $k \ge 2$ and $W \ge 2$. Since the number of states of process $\underline{X}_k(t)$ grows very fast with the number k of hops in the path and the number W of wavelengths, it is not possible to obtain the call-blocking probabilities directly from the above process. Consequently, an approximate model was constructed in[14] to analyze a single-class k-hop path. The approximation consists of modifying the call arrival process to obtain a time-reversible Markov process that has a closed-form solution. To illustrate, let us consider the Markov process corresponding to a two-hop path $\underline{X}_2(t) = (N_{11}(t), N_{12}(t), N_{22}(t), F_{12}(t))$. We now modify the arrival process of calls that use both hops (a Poisson process with rate λ_{12} in the exact model) to a state-dependent Poisson process with rate Λ_{12} given by

$$\Lambda_{12}(n_{11}, n_{12}, n_{22}, f_{12}) = \lambda_{12}\frac{f_{12}(W-n_{12})}{f_{11}f_{22}},$$
$$f_{ii} = W - n_{ii}. \tag{2}$$

The arrival process of other calls remain as in the original model. As a result, we obtain a new Markov process $\underline{X}'_2(t)$ with the same state space and the same state transitions as process $\underline{X}_2(t)$, but which differs from the latter in some of the state transition rates. The intuition for the modification shown in expression (2) can be found in [14].

Under the new arrival process, the Markov process $\underline{X}'_2(t)$ is time-reversible and the stationary vector π is given by

$$\pi(n_{11}, n_{12}, n_{22}, f_{12}) = \frac{1}{G_2(W)}\frac{\rho_{11}^{n_{11}}}{n_{11}!}\frac{\rho_{12}^{n_{12}}}{n_{12}!}\frac{\rho_{22}^{n_{22}}}{n_{22}!} \times \frac{\binom{f_{11}}{f_{12}}\binom{n_{11}}{W-n_{12}-n_{22}-f_{12}}}{\binom{W-n_{12}}{W-n_{12}-n_{22}}} \tag{3}$$

where $G_k(W)$ is the normalizing constant for a k-hop path with W wavelengths. Let $P(n_{11}, n_{12}, n_{22})$ be the marginal distribution over the states for which $N_{ij}(t) = n_{ij}, 1 \le i \le j \le 2$. It can be verified [14] that $P(n_{11}, n_{12}, n_{22}) = (1/G_2(W))(\rho_{11}^{n_{11}}/n_{11}!)(\rho_{12}^{n_{12}}/n_{12}!)(\rho_{22}^{n_{22}}/n_{22}!)$. Likewise, for a k-hop path, $k \ge 2$, with the modified state-dependent Poisson arrival process, the marginal distribution over the states for which $N_{ij}(t) = n_{ij}, 1 \le i \le j \le k$, is given by

$$P(n_{11}, n_{12}, \ldots, n_{kk}) = \frac{1}{G_k(W)}\prod_{\{(i,j)\,|\,1\le i\le j\le k\}}\frac{\rho_{ij}^{n_{ij}}}{n_{ij}!}. \tag{4}$$

It is easily seen that this distribution is the same as in the case of a network with wavelength converters at each node. An interesting feature of having wavelength converters at every node is that the network has a product-form solution even when there are multiple classes of calls on each route, as long as call arrivals are Poisson and holding times are exponential [4]. Further, when calls of all classes are assigned wavelengths from the same set, we can aggregate classes to obtain an equivalent single-class model with the same steady-state probability distribution over the aggregated states, as we show next.

B. The Multiclass Single-Path Network

Let us now consider a k-hop path with wavelength converters at all nodes, and with R classes of calls. If $\lambda_{ij}^{(r)}, 1 \le i \le j \le k, 1 \le r \le R$ is the arrival rate of calls of class r on route (i,j), and $1/\mu_{ij}^{(r)}, 1 \le i \le j \le k, 1 \le r \le R$ is the mean of the exponential holding time of calls of class r, the probability of being in state $\underline{n} = (n_{11}^{(1)}, n_{11}^{(2)}, \ldots, n_{11}^{(R)}, n_{12}^{(1)}, \ldots, \ldots, n_{kk}^{(R)})$ is

$$P(\underline{n}) = \frac{1}{G_k(W)}\left(\prod_{\{(i,j)\,|\,1\le i\le j\le k\}}\prod_{r=1}^{R}\frac{\left(\rho_{ij}^{(r)}\right)^{n_{ij}^{(r)}}}{n_{ij}^{(r)}!}\right). \tag{5}$$

Let $\sigma_{ij} = \sum_r \rho_{ij}^{(r)}$ and $s_{ij} = \sum_r n_{ij}^{(r)}$. s_{ij} is the total number of calls of all classes that use segment (i,j) of the path, and σ_{ij} is the total offered load of these calls. Taking the summation of (5) over all states such that $\sum_r n_{ij}^{(r)} = s_{ij}, 1 \le i \le j \le k$, we obtain

$$P'(s_{11}, s_{12}, \ldots, s_{kk}) = \sum_{\{\underline{n}\,|\,\sum_r n_{ij}^{(r)} = s_{ij}\}} P(\underline{n})$$
$$= \frac{1}{G_k(W)}\prod_{\{(i,j)\,|\,1\le i\le j\le k\}}\frac{\sigma_{ij}^{s_{ij}}}{s_{ij}!}. \tag{6}$$

Observe that this is identical to the solution (4) for the single-class case obtained by substituting σ_{ij} by ρ_{ij} and s_{ij} by n_{ij} in (6). We conclude that by employing class aggregation on a multiclass path with converters at all nodes, we obtain a system equivalent to a single-class path with converters. In Section III-A, we showed that the modified single-class wavelength-routing network without converters has a steady-state marginal distribution similar to the exact single-class network with converters. We now show that a modified multiclass network without wavelength converters can also be subjected to class aggregation to obtain an equivalent single-class model. The modification applied to the arrival process of calls is similar to the single-class case, and it is given by

$$\Lambda_{ij}^{(r)}(\underline{x}) = \lambda_{ij}^{(r)} f_{ij}\frac{\left(\sum_{l=1}^{i} s_{li} + f_{ii}\right)}{f_{ii}} \times \frac{\left(\sum_{l=1}^{i+1} s_{l(i+1)} + f_{(i+1)(i+1)}\right)}{f_{(i+1)(i+1)}}\cdots \times \frac{\left(\sum_{l=1}^{j-1} s_{l(j-1)} + f_{(j-1)(j-1)}\right)}{f_{(j-1)(j-1)}}. \tag{7}$$

Then, by summing up the probabilities of all states of the modified multiclass network such that $\sum_r n_{ij}^{(r)} = s_{ij}$, the probability that the equivalent single-class network without converters is in state $S_{\underline{x}} = (s_{11}, s_{12}, \ldots, s_{1k}, s_{22}, \ldots, s_{kk}, f_{12}, f_{13}, \ldots, f_{(k-1)k})$ is given by

$$\pi(S_{\underline{x}}) = \frac{1}{G_k(W)}\left(\prod_{i,j}\frac{s_{ij}!}{\sigma_{ij}^{s_{ij}}}\right)\left\{\prod_{l=2}^{k}\binom{f_{1(l-1)}}{f_{1l}} \times \frac{\prod_{m=2}^{l-1}\binom{f_{m(l-1)} - f_{(m-1)(l-1)}}{f_{ml} - f_{(m-1)l}}}{\binom{f_{ll}+n_{ll}}{f_{ll}}} \times \binom{f_{ll}+n_{ll}-f_{(l-1)(l-1)}}{f_{ll}-f_{(l-1)l}}\right\}. \tag{8}$$

C. Blocking Probabilities in the Multiclass Case

Since the arrival rate of calls of each class on each route is Poisson, the blocking probability, $Q_{ij}^{(r)}$, of a call of class r using route (i,j) is just the fraction of time that there are no wavelengths that are free on all hops along route (i,j) (see the *PASTA* theorem in [12]). Thus, $Q_{ij}^{(r)} = \lim_{\tau\to\infty}(\int_{t=0}^{\tau} I_{\{F_{ij}(t)=0\}}\,dt/\tau)$, where

$I_{\{F_{ij}(\tau)=0\}} = 1$, if $F_{ij}(\tau) = 0$, and zero otherwise. As can be seen, the blocking probability is class-independent.

Next, we focus on the call-blocking probabilities in the modified model. The arrival process of calls of class r on route (i, j) is a state-dependent Poisson process whose rate at time τ, $A_{ij}^{(r)}(\tau)$ is a function of the state $\underline{X}(\tau)$ of the process and is given by

$$A_{ij}^{(r)}(\tau) = \Lambda_{ij}^{(r)}(\underline{X}(\tau)) = \lambda_{ij} \frac{F_{ij}(\tau) \prod_{k=i}^{j-1} \left(\sum_{l=1}^{k} N_{lk}(\tau) + F_{kk}(\tau) \right)}{F_{ii} F_{i+1,i+1} \dots F_{jj}}. \quad (9)$$

The modified arrival process satisfies: $\Lambda_{ij}^{(r_1)}(\underline{x})/\Lambda_{ij}^{(r_2)}(\underline{x}) = \lambda_{ij}^{(r_1)}/\lambda_{ij}^{(r_2)}, 1 \le r_1, r_2 \le r$. By applying the *PASTA* theorem conditioned on being in state $\underline{x}$, the conditional call blocking probability, $\mathcal{P}_{ij}^{(r)}(\underline{x})$, of calls of class r on route (i, j) is given by the fraction of time spent in state $\underline{x}$ in which there are no wavelengths that are free on all hops of route (i, j). Therefore

$$\mathcal{P}_{ij}^{(r)}(\underline{x}) = \lim_{\tau \to \infty} \frac{\int_{t=0}^{\tau} I_{\{F_{ij}(t)=0, \underline{X}(t)=\underline{x}\}}\, dt}{\int_{t=0}^{\tau} I_{\{\underline{X}(t)=\underline{x}\}}\, dt} = \begin{cases} 1, & \text{if } f_{ij} = 0 \\ 0, & \text{otherwise} \end{cases} \quad (10)$$

Let $P_{ij}^{(r)}$ be the unconditional probability that a call of class r, on route (i, j) gets blocked in the modified multiclass model. This is given by

$$P_{ij}^{(r)} = \frac{\sum_{\underline{x}} \Lambda_{ij}^{(r)}(\underline{x}) \pi(\underline{x}) \mathcal{P}_{ij}^{(r)}(\underline{x})}{\sum_{\underline{x}} \Lambda_{ij}^{(r)}(\underline{x}) \pi(\underline{x})} = \frac{\sum_{\{\underline{x} f_{ij}=0\}} \Lambda_{ij}^{(r)}(\underline{x}) \pi(\underline{x})}{\sum_{\underline{x}} \Lambda_{ij}^{(r)}(\underline{x}) \pi(\underline{x})} \quad (11)$$

and is also independent of the class r. Thus, by computing the blocking probability on the equivalent single-class path, we can obtain the solution to the multiclass path.

D. Multiclass Mesh Networks

The solution to single-class networks with a mesh topology, wavelength converters at an arbitrary subset of nodes, and static or fixed alternate routing, has been presented in [14]. This solution involves decomposition of the network into short path segments with two or three hops, and analyzing these approximately using expression (3), or the corresponding expression for a three-hop path which can be found in [14]. The solutions to individual segments are appropriately combined to obtain a value for the blocking probability of calls that traverse more than one segment. The effect of the wavelength continuity requirement is captured by an approximate *continuity factor* that is used to increase the blocking probability of calls continuing to the next segment to account for the possible lack of common free wavelengths in the two segments (for details on the continuity factor see [14]). The process repeats until the blocking probabilities converge. By applying class aggregation, the same algorithms may be used to calculate blocking probabilities for multiclass networks. Specifically, we use these steps for a network with R classes of calls.

1) **Path decomposition:** Decompose the multiclass mesh network topology into single-path subsystems using the algorithm in [14].
2) **Time-reversible process approximation:** For each single-path subsystem, modify the arrival process as in (7) to obtain an approximate time-reversible Markov process.
3) **Class aggregation:** For each path subsystem, apply class aggregation to obtain an equivalent single-class subsystem.
4) **Calculation of blocking probabilities:** For each subsystem, obtain the blocking probabilities as follows. If the path is at most three hops long, use (11) and (8) directly. If the path is longer than three hops, decompose it into two- or three-hop paths which are solved in isolation, and combine the solutions to obtain the blocking probabilities along the original longer path (see [14]).
5) **Convergence:** Repeat Steps 2 to 4, after appropriately modifying the original arrival rates to each single-path subsystem to account for the new values of the blocking probabilities obtained in Step 4 (see [14]), until the blocking probabilities converge.

IV. Multiclass Networks With Multicast Calls

A. Multicast Calls With Poisson Arrivals

Let us now consider a network employing optical splitters which can carry both unicast and multicast calls. A small set of predetermined multicast trees is used for routing each multicast call, and the call is blocked if no free wavelength is found in any of the trees. Consider a multicast call from i to a set of destination nodes $V = \{j_1, j_2, \dots, j_v\}$. There are two possibilities. First, i and all the destination nodes may lie on a single linear path. Let j_m and j_n be the nodes at the extremities of this path. For purposes of evaluating call blocking probabilities, this multicast session is equivalent to a unicast call between node j_m and j_n. Let λ_{j_m,j_n} be the Poisson arrival rate of unicast calls between nodes j_m and j_n, $1/\mu_{j_m,j_n}$ be the mean of their exponential holding time, and $\rho_{j_m,j_n} = \lambda_{j_m,j_n}/\mu_{j_m,j_n}$. Let $\lambda_{i,V}$ be the Poisson arrival rate of multicast calls from source node i to a set of destination nodes V, $1/\mu_{i,V}$ be the mean of their exponential holding time and $\rho_{i,V} = \lambda_{i,V}/\mu_{i,V}$. Then, the given multicast network can be analyzed as a unicast network with the following simple modification: $\rho_{j_m,j_n}^{(new)} = \rho_{j_m,j_n} + \rho_{i,V}$. The resulting network may be analyzed using our path decomposition algorithm.

In general, the source and destination nodes need not lie on a single path. Even in this case, the multicast tree may be broken up into several linear segments, such that each segment starts at the source (or a branch node) and terminates at a branch node (or a destination node). This network can be also analyzed using our path decomposition algorithm by considering each segment of the tree as a subsystem. In turn, each subsystem may be analyzed as a unicast network after applying the modification above for each multicast group using the subsystem. The blocking probability of a multicast call can then be expressed in terms of the

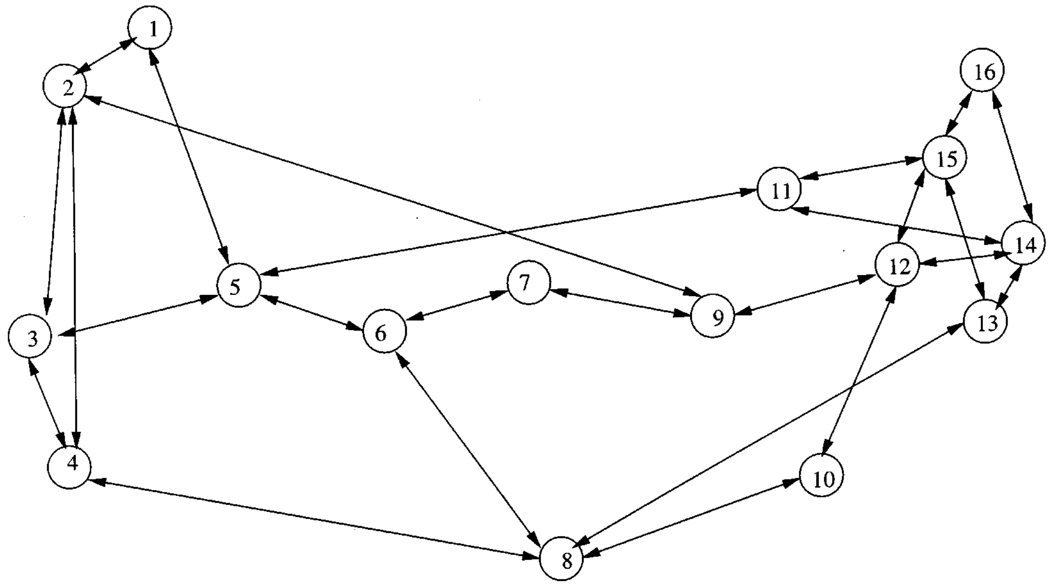

Fig. 2. The NSF network.

blocking probability along the segments to which its tree has been decomposed.

B. Multicast Calls With State-Dependent Poisson Arrivals

In practice, it is unlikely that there will be several concurrently active multicast sessions among the same set of source and destination nodes. To account for this fact, we now consider a state-dependent Poisson arrival model for multicast calls. Let us assume that there may be at most one active multicast session for each group, defined by the ordered pair $G_i = (i, V)$, where i is the source node and V is the set of destination nodes. The ordered pair G_i may also be used to represent the route of the multicast group. Likewise, the route of a unicast call from node i to node j is denoted by the ordered pair (i, j). Let $\underline{R}$ represent the total number of routes in the network for both unicast and multicast calls.

We first consider the single-class case. Let $\lambda_{(i,j)}$ be the Poisson arrival rate of calls on a unicast route. Let $(1/\mu_{(i,j)})$ be the mean of the exponential holding time. Let $(1/\lambda_{G_i})$ be the mean of the exponentially distributed time until the arrival of a multicast call to group G_i, conditioned on there being no active sessions. The arrival process of multicast calls is, therefore, state-dependent Poisson. Let $1\mu_{G_i}$ be the mean of the exponential holding time. We use a Poisson approximation to obtain the blocking probabilities of this type of network, which we now proceed to describe.

Suppose the wavelength-routing network has an unlimited number of wavelengths. Then, the state of the network may just be defined by the number of active calls on each route, including multicast routes. Let $N_{\underline{r}}(t)$ be the number of active calls on route $\underline{r}$ at time t. The state of the system at time t may, therefore, be represented by $S(t) = (N_1(t), N_2(t), \ldots, N_{\underline{R}}(t))$. Let $\pi(\underline{n})$ represent the probability $P(S(t) = \underline{n})$ in steady state. When there is an unlimited number of wavelengths, the Markov chain $S(t)$ has a product-form solution, and $\pi(\underline{n})$ may be written as: $\pi(n_1, n_2, \ldots, n_{\underline{R}}) = \prod_{\underline{r}=1}^{\underline{R}} P_{\underline{r}}(n_{\underline{r}})$, where $P_{\underline{r}}(n_{\underline{r}})$ is the marginal probability that the number of active calls on route $\underline{r}$ is $n_{\underline{r}}$ in steady state. If $\underline{r}$ is a unicast route, $P_{\underline{r}}(n_{\underline{r}})$ is given by: $P_{\underline{r}}(n_{\underline{r}}) = (1/n_{\underline{r}}!) \times (\lambda_{\underline{r}}/\mu_{\underline{r}})^{n_{\underline{r}}} \times \exp(-\lambda_{\underline{r}}/\mu_{\underline{r}})$. For multicast routes, we have: $P_{\underline{r}}(0) = \mu_{\underline{r}}/(\mu_{\underline{r}} + \lambda_{\underline{r}}), P_{\underline{r}}(1) = \lambda_{\underline{r}}/(\mu_{\underline{r}} + \lambda_{\underline{r}})$, and $P_{\underline{r}}(n_{\underline{r}}) = 0$ if $n_{\underline{r}} > 1$. Thus, the mean arrival rate of calls on multicast route $\underline{r}$ is given by $\lambda_{\underline{r}}\mu_{\underline{r}}/(\mu_{\underline{r}} + \lambda_{\underline{r}})$. We substitute this to obtain an approximate state-independent Poisson arrival model for multicast calls. Thus, the arrival rate of calls to multicast route $\underline{r}$ is assumed Poisson with mean $\Lambda_{\underline{r}} = \lambda_{\underline{r}}\mu_{\underline{r}}/(\mu_{\underline{r}} + \lambda_{\underline{r}})$. Given these Poisson arrivals, we can use the modification in Section IV.A to obtain an equivalent single-class network. With a multiclass network, we first apply the Poisson approximation to each class of multicast calls with state-dependent Poisson arrivals. From this, an equivalent single-class network may be constructed in the same way as shown in Section III-B. Then, the modification in Section IV-A is applied to this single-class network.

TABLE I
ARRIVAL AND SERVICE PARAMETERS FOR THE NSF NETWORK

	Class 1	Class 2	Class 3
λ	0.1	0.2	0.15
μ	3.0	6.0	4.0

V. NUMERICAL RESULTS

In this section, we validate our approximate algorithm for multiclass networks. First, we consider a network with unicast traffic only and a topology similar to that of the NSF network, shown in Fig. 2. There are 16 nodes and 240 unidirectional routes. There are three classes of calls on each route. The arrival and service rates of calls of a particular class are the same on each route and are shown in Table I. The blocking probabilities are plotted in Fig. 3 for four routes, as a function of the

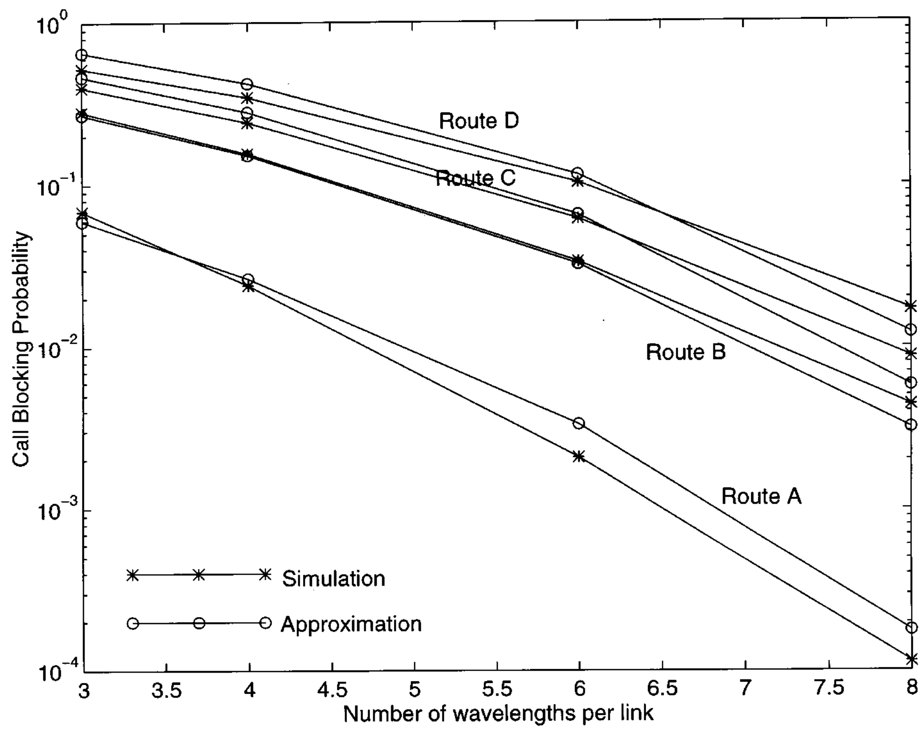

Fig. 3. Blocking probabilities for the NSF network.

number of wavelengths on each link. Route A is a single-hop route from node 1 to node 5. Route B has two hops, connecting node 1 to node 3 via node 2. Route C has three hops, connecting node 1 to node 4 via nodes 2 and 3. Route D has four hops, connecting node 4 to node 5 via nodes 3, 2 and 1.

From Fig. 3 we can see that the blocking probability decreases as W increases, as expected. We also observe that the length of the path used by a call considerably affects the blocking probability experienced by the call. Specifically, for a given number W of wavelengths, the blocking probability increases with the number of hops in a route, such that calls on Route A (a single-hop path) have the lowest blocking probability while calls on Route D (a four-hop path) the highest. Most importantly, we can see that there is good agreement between the analytical and simulation results. The results indicate that our approximate method can be used to estimate accurately the blocking probabilities for all calls in the network.

We also consider the 3×3 torus network in Fig. 4 with multicast calls. There are 18 one-hop unicast routes and 18 two-hop routes, for a total of 36 unicast routes. There are six multicast trees, each of which may be listed by the set of links they consist of: $\{1,5,6\}, \{2,4,7\}, \{3,1,10\}, \{3,10,12\}, \{6,10,13,14\}$, and $\{8,13,15\}$. We assume these constitute routes 37 to 42. Each multicast route is assumed to carry at most two concurrent multicast calls. There are three classes of calls (unicast or multicast) on each route. The arrival rate of calls on each unicast route was set to 0.4, 0.6, and 0.5 calls per time unit for Class 1, Class 2, and Class 3, respectively. The arrival rate of calls on each multicast route, conditioned on there being no existing multicast calls was set to 0.4, 0.6, and 0.5, for the three

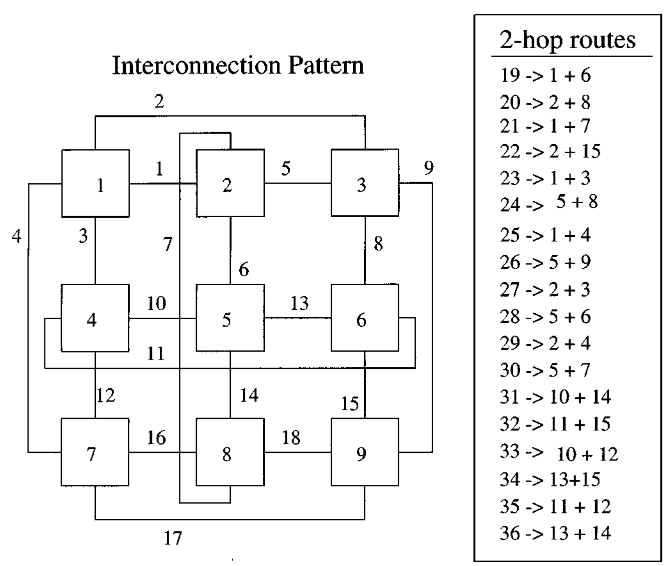

Fig. 4. The 3×3 torus network.

classes. Also, the mean holding time was set equal to $1/3, 1/5$, and $1/6$ time units, respectively.

In Fig. 5, we plot the call blocking probabilities obtained through the approximation method and through simulation, assuming random wavelength allocation. The results obtained by the approximate analysis are in close agreement to simulation results for blocking probability values ranging from very high (e.g., around 10^{-1}) to very low (around 10^{-5}). We also note that both the *relative* and the *absolute* differences in the blocking

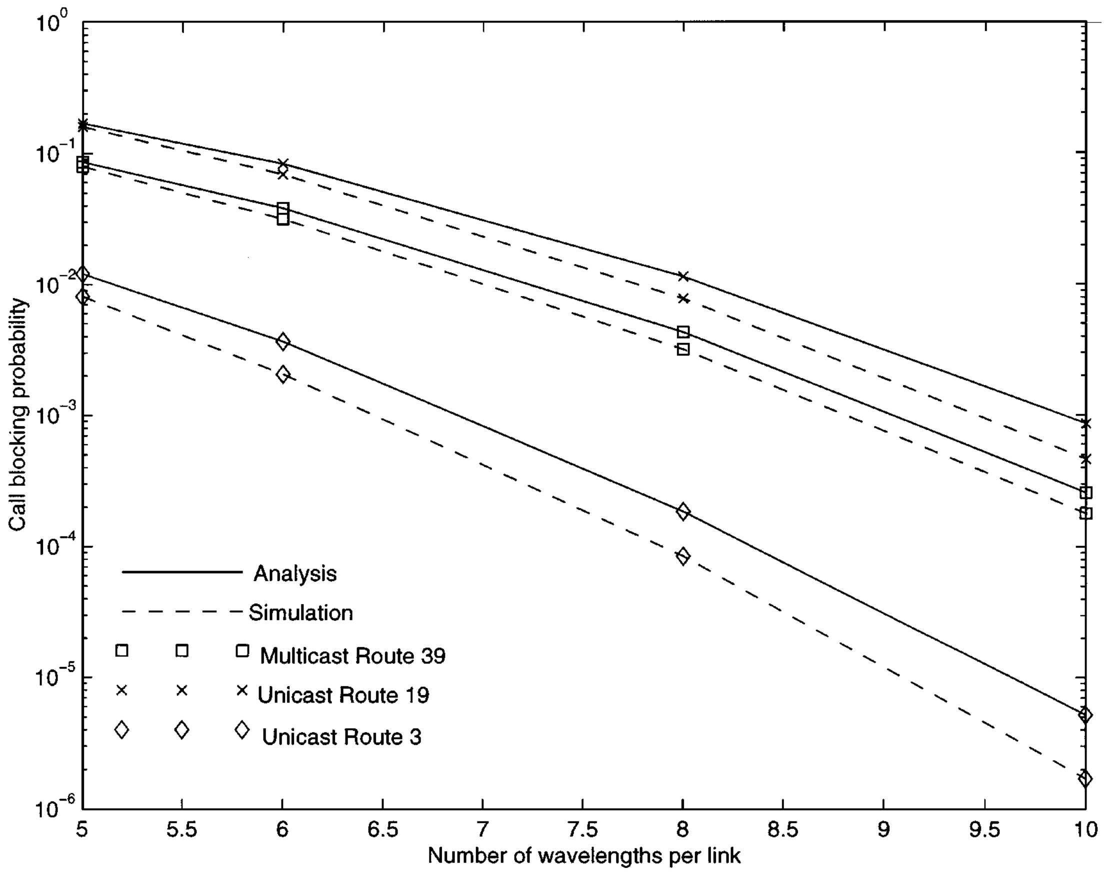

Fig. 5. Call blocking probability in torus multicast network.

probability values computed by the analysis and the simulation are small, but the relative difference is larger for very low blocking probability values as expected.

VI. Concluding Remarks

We have considered the problem of computing call blocking probabilities in multiclass multicast wavelength-routing networks which employ the random wavelength allocation policy. Our approach consists of modifying the call arrival process to obtain an approximate multiclass network model with only point-to-point calls, using class aggregation to map this to an equivalent single-class network and employing path decomposition algorithms on the latter to determine the call blocking probabilities. Results similar to the ones presented here have been obtained for a wide range of traffic loads and different classes of calls, and for other network topologies. Overall, our work demonstrates that path decomposition can be a practical and powerful technique, allowing us to analyze networks with multiple classes of point-to-point and multicast traffic.

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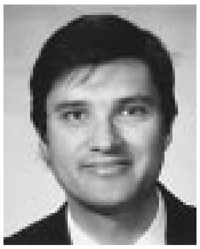

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