## Optimal Granularity of MPLS Tunnels *

George N. Rouskas and Laura E. Jackson
Department of Computer Science, North Carolina State University, Raleigh, NC 27695-7534, USA
E-mail: {rouskas,lejacks2}@csc.ncsu.edu

We consider the problem of tunneling MPLS label switched paths (LSPs) over an optical network in which $\lambda$-channels (lightpaths) of a fixed capacity are the main mechanism of transport. While any set of LSPs, each LSP having an arbitrary rate, can be tunneled over a given $\lambda$-channel as long as the aggregate rate does not exceed the channel's capacity, intuition suggests that this flexibility has a limit. In other words, we believe that it is sufficient for the network to provide a small set of fixed service levels in terms of the LSP data rate. We present an efficient dynamic programming algorithm to obtain an optimal set of service levels given (a) a fixed set of LSPs and corresponding bandwidth requests, or (b) the probability distribution of LSP bandwidth requests. The set of optimal service levels strikes a balance between the conflicting goals of simplicity and performance. Numerical results indicate that with as few as ten service levels, the performance penalty compared to a continuous-rate network is less than 10% of the total amount of bandwidth requested for a wide variety of LSP data rate distributions.

### 1. Introduction

Over the last few years we have witnessed a wide deployment of wavelength division multiplexing (WDM) technology in the Internet infrastructure, and the creation of optical backbone networks. The main mechanism of transport in such a network is the $\lambda$-channel (lightpath), which is a communication channel established between two nodes over a network of optical cross-connects (OXCs), and which may span a number of fiber links (physical hops). The Internet Engineering Task Force (IETF) is investigating the use of Generalized MPLS (GMPLS) [1] to set up and tear down $\lambda$-channels. GMPLS is an extension of MPLS [2] that supports multiple types of switching, including switching based on wavelengths usually referred to as Multi-Protocol Lambda Switching (MP$\lambda$S).

With currently available optical technology, the data rate of each $\lambda$-channel is in the order of 2.5-10 Gbps, while channels operating at 40 Gbps will be commercially available in the near future. In order to utilize efficiently the capacity of each $\lambda$-channel, service providers will need to aggregate a number of lower-rate traffic streams into a single $\lambda$-channel. The GMPLS standard makes it possible to tunnel a set of MPLS label-switched paths (LSPs), with the same originating and terminating nodes, by carrying them within the same $\lambda$-channel.

It is the interface between MPLS networks and optical backbones of OXCs that is the subject of study in this paper. The MPLS packet-switching technology can support LSPs with data rates ranging from a few Kbps to several Gbps; in theory, the range of supported data rates is infinite. Optical WDM networks, on the other hand, are circuit-switched and the capacity of each $\lambda$-channel is fixed. The GMPLS standard is flexible in that any set of LSPs, each LSP operating at an arbitrary data rate, may be aggregated into a $\lambda$-channel as long as the aggregate rate does not exceed the capacity of the channel. Intuition, however, suggests that this flexibility has a limit, and that the number of service levels offered by the MPLS network does not have to be infinite.

Operating a network that provides only a (small) set of quantized service levels makes sense in many respects. In particular, many functions including traffic engineering, packet scheduling and QoS support, network management, traffic policing, and billing are significantly simplified compared to a network offering a continuous spectrum of data rates. Performance analysis is also more tractable, since networks with continuous rates give rise to analytical models with infinite dimensions (note also that these models

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are usually approximated by finite-dimensional ones). On the other hand, limiting the number of supported data rates does have a disadvantage in that it may use more bandwidth than a continuous-rate network to accommodate a given set of customer requests. Specifically, a request for service, rather than receiving the exact data rate needed, may have to subscribe to the next highest data rate offered by the network. As a result, bandwidth quantization will have an adverse effect on network performance which will manifest itself either as a higher blocking probability (i.e., a higher probability of denying a request for service compared to a continuous-rate network employing the same number of $\lambda$-channels) or as a lower utilization (since a larger number of $\lambda$-channels may need to be employed to carry the same set of service requests as a continuous-rate network).

The goal of this paper is to determine the set of service levels (i.e., the bandwidth granularity of MPLS tunnels) that strikes a balance between the two conflicting goals of simplicity and performance. Specifically, we develop polynomial time algorithms to compute the optimal network service levels given information about the customer demands. Therefore, our algorithms will allow network providers to tailor the services they provide to the needs of their IP customers.

We note that, while our work is motivated by the emerging optical WDM networks, the problem we study is inherent in any network environment in which a number of lower rate streams are multiplexed for transport over a higher rate channel. However, the problem takes on new significance in optical networks, since a single optical channel is capable of carrying hundreds, even thousands, of independent LSPs. Managing and controlling such a large number of traffic streams at Gigabit per second data rates is an extremely complicated task, while the complexity of certain functions (e.g., QoS scheduling) increases faster than linearly with the number of LSPs. Optimal bandwidth quantization techniques, such as the ones presented in this paper, have the potential to simplify the network operation, to enable service providers to offer a menu of optimized services, and to make it easier to guarantee these services.

This paper is organized as follows. Section 2 discusses the quantized MPLS network under study. Section 3 presents a dynamic programming algorithm for computing the optimal service levels for a set of known traffic demands. Section 4 extends the dynamic programming algorithm to the situation where only the distribution of the size of the traffic demands is known. We then conclude the paper in Section 5.

## 2. The Quantized MPLS Network

We consider a network of OXCs that provides interconnection to a number of edge nodes. The services that the OXC network offers are in the form of *logical* connections (tunnels) implemented using $\lambda$-*channels*. We let $C$ denote the capacity of each $\lambda$-channel. The network provides $L \geq 1$ quantized levels of service. The $l$-th level of service corresponds to tunnels of bandwidth $b_l$, such that $b_{l-1} < b_l, l = 1, \cdots, L$. For notational convenience we also define $b_0 = 0$ (the 0-th level of service).

A request for service represents a request to establish a label switched path (LSP) between two nodes interconnected by the optical network. An LSP request is fully specified by the two endpoints and its bandwidth demand $d$. An LSP request can be satisfied if there is sufficient capacity in an existing $\lambda$-channel between the specified endpoints to carry the corresponding traffic, or if a new $\lambda$-channel with sufficient capacity can be set up. If the request can be satisfied, it is assigned to service level $l$ such that $b_{l-1} < d \leq b_l$. In other words, the LSP is carried over a tunnel with the minimum capacity to accommodate its traffic. Let $d_{max}$ denote the maximum bandwidth that can be requested, $d_{max} \leq C$. In order to satisfy the maximum request, we must set $b_L = d_{max}$.

We note that the bandwidth quantization problem we consider in this work has applications to the service level agreements (SLAs) between the optical network provider and its customers. In particular, the bandwidth demand $d$ of an LSP setup request may correspond to the peak rate of the LSP traffic, the average rate, or some value between the two. Similarly, the bandwidth of a service level (tunnel) can be simply thought of as a committed rate. Any excess bandwidth within the $\lambda$-channel may be distributed in some arbitrary fashion (specified in the SLAs) among active LSPs by employing an appropriate scheduling algorithm at the node which performs aggregation of the LSPs for transport over the optical network. The node performing the aggregation may also employ policing, shaping, or scheduling functions to provide QoS guarantees to individual LSPs as specified in the SLAs. The point is that bandwidth quantization does not require any changes in the deployed infrastructure of router functions dealing with QoS, policing, traffic engineering, billing, etc. Instead, quantization *simplifies* the management and control of this infrastructure since the network only has to provide a (small) set of discrete, well-known service levels, rather than having to support a continuous range of unpredictable traffic demands.

## 3. Deterministic Set of Service Requests

In this section we assume that the set of service requests (i.e., the endpoints and the bandwidth demand of each LSP) is fixed and known in advance. Our objective is twofold. First, for a given $L$ we wish to determine the optimal set of $L$ service levels for the known set of service requests. Second, we wish to determine the performance penalty for an optimally quantized network, compared to a continuous-rate network, as a function of the number of service levels $L$. We note that LSPs with different endpoints are tunneled over different $\lambda$-channels. Therefore, without loss of generality, for the remainder of this section we only consider LSP requests with the same origin and destination nodes.

### 3.1. The Deterministic Optimization Problem

Consider a fixed set $\mathcal{R}$ of $N$ service requests, $\mathcal{R} = \{r_i, i = 1, \cdots, N\}$, for the establishment of $N$ LSPs between the same (directed) pair of nodes. Let $d_i$ denote the bandwidth demand of request $r_i$. Without loss of generality, we assume that the requests are sorted in non-decreasing order of their traffic demands, i.e., that: $d_1 \leq d_2 \leq \cdots \leq d_{N-1} \leq d_N$. We let $D(\mathcal{R})$ be the total bandwidth demand: $D(\mathcal{R}) = \sum_{i=1}^{N} d_i$. $D(\mathcal{R})$ is the amount of bandwidth needed to carry all $N$ LSPs in a continuous-rate network.

Let $\mathcal{S}_L$ be a set of $L$ quantized service levels, $\mathcal{S}_L = \{b_i, i = 1, \cdots, L\}$. Let $C(\mathcal{R}, \mathcal{S}_L)$ denote the amount of bandwidth required to satisfy all the $N$ service requests in set $\mathcal{R}$ when the network offers the set of service levels $\mathcal{S}_L$. We have that:

$$C(\mathcal{R}, \mathcal{S}_L) = \sum_{l=1}^{L} k_l b_l \tag{1}$$

where $k_l$ is the set of requests in $\mathcal{R}$ with demands strictly greater than $b_{l-1}$ but no more than $b_l$. Since some of the LSPs may receive a level of service higher than that requested, we have that $C(\mathcal{R}, \mathcal{S}_L) \geq D(\mathcal{R})$. The additional bandwidth required $C(\mathcal{R}, \mathcal{S}_L) - D(\mathcal{R})$ is the performance penalty associated with the quantization process.

Our objective is to minimize the performance penalty in order to minimize the additional cost (in terms of bandwidth) required to support a given set of requests (alternatively, to minimize the amount of foregone revenue due to the denial of some of the service requests when additional capacity cannot be provided). Since $D(\mathcal{R})$ is constant for a given request set $\mathcal{R}$, the optimization problem can be expressed as follows:

Given a set of demands $\mathcal{R}$ and an integer $L$, find the set of $L$ service levels that minimizes (1).

We let $\mathcal{S}_L^\star$ denote the optimal set of $L$ service levels, and $C^\star(\mathcal{R}, L)$ denote the amount of bandwidth required to carry all requests in $\mathcal{R}$ under this optimal set of service levels. The following properties of $\mathcal{S}_L^\star$ and $C^\star(\mathcal{R}, L)$ are straightforward to verify.

**P1**. $\mathcal{S}_1^\star = \{d_N\}$, $C^\star(\mathcal{R}, 1) = N d_N$.

**P2**. $\mathcal{S}_N^\star = \mathcal{R}$, $C^\star(\mathcal{R}, N) = D(\mathcal{R})$.

**P3**. $\mathcal{S}_M^\star = \mathcal{R} \bigcup A$, $C^\star(\mathcal{R}, M) = D(\mathcal{R})$, $A$ a set of $k \geq 1$ arbitrary service levels, $M = N + k$.

**P4**. $C^\star(\mathcal{R}, L) \geq C^\star(\mathcal{R}, L+1)$, $L = 1, 2, \cdots$.

Property **P1** is a consequence of our remark in Section 2 that, in order to satisfy the maximum traffic demand (i.e., $d_N$), the highest service level (i.e., $b_1$) must equal this maximum demand. Property **P2** simply states that, if the number of service levels is equal to the traffic demands, the optimal strategy is for the network to simply provide the requested levels of traffic demands; in this case, the amount of bandwidth required is the same as in a continuous-rate network, and there is no performance penalty due to quantization. Property **P3** implies that there is no advantage in offering a number of service levels greater than the number of service requests. (Note that if there are only $n \leq N$ distinct levels of traffic demands, properties **P2** and **P3** can be strengthened by using $n$ in the place of $N$.) Finally, property **P4** states that offering more service levels reduces the performance penalty due to quantization. Of course, properties **P2** and **P3** show that there is a limit to how many service levels the network should offer; however, this limit (the number $N$ of service requests, or the number $n \leq N$ of distinct service demands) is rather loose. One issue we address in this section is to determine the incremental benefit of offering an additional level of service in order to obtain a tighter practical limit on the number of offered service levels.

The following lemma generalizes properties **P1** and **P2**, and provides the basis for the dynamic programming algorithm developed in the next subsection.

**Lemma 3.1** *If $L \leq N$, the $L$ optimal service levels coincide with $L$ service demands, i.e., $\mathcal{S}_L^\star \subset \mathcal{R}$.*

**Proof.** By contradiction. Suppose that $\mathcal{S}$ is an optimal set of $L$ service levels for request set $\mathcal{R}$, $L \leq N$, and that some service level $b_l \in \mathcal{S}$ is not equal to the bandwidth $d_i$ of some request $r_i \in \mathcal{R}$. There are two cases to consider.
*Case 1.* There exists at least one request $r_m$ with demand $d_m$ such that $b_{l-1} < d_m < b_l$. Let $r_n$ be the request with the highest demand $d_n$ among all such requests $r_m$. It is easy to see that the new set of $L$ service levels $\mathcal{S}' = (\mathcal{S} \bigcup \{d_n\}) - \{b_l\}$ is such that $C(\mathcal{R}, \mathcal{S}') \leq C(\mathcal{R}, \mathcal{S})$. Therefore, $\mathcal{S}$ cannot be an optimal set of service levels.
*Case 2.* There is no request $r_m$ with demand $d_m$ such that $b_{l-1} < d_m < b_l$. Since $L \leq N$, there is at least one request $r_n$ with demand $d_n \neq b_i, i = 1, \cdots, L$. Under $\mathcal{S}$, request $r_n$ receives a higher service than requested. Again, it is straightforward to see that the new set of $L$ service levels $\mathcal{S}'' = (\mathcal{S} \bigcup \{d_n\}) - \{b_l\}$ is such that $C(\mathcal{R}, \mathcal{S}'') \leq C(\mathcal{R}, \mathcal{S})$. Therefore, $\mathcal{S}$ cannot be an optimal set of service levels. ■

### 3.2. Dynamic Programming Algorithm

Recall that we have assumed that the requests $r_i \in \mathcal{R}$ are sorted in non-decreasing order of their traffic demands $d_i$. In order to describe the dynamic programming algorithm for finding an optimal set of $L$ service levels, we define $\mathcal{R}_n$ as the set of service requests from $\mathcal{R}$ with the $n$ smallest traffic demands: $\mathcal{R}_n = \cup_{i=1}^{n} \{r_i\}, n = 1, \cdots, N$. Note that $\mathcal{R} = \mathcal{R}_N$. $C^\star(\mathcal{R}, L)$ can now be computed recursively as:

$$C^\star(\mathcal{R}_n, 1) = n d_n, \;\; n = 1, \cdots, N \tag{2}$$

$$C^\star(\mathcal{R}_1, l) = d_1, \;\; l = 1, \cdots, L \tag{3}$$

$$C^\star(\mathcal{R}_n, l+1) = \min_{q=l,\cdots,n-1} \left\{ C^\star(\mathcal{R}_q, l) + (n-q) d_n \right\}, \quad l = 1, \cdots, L-1, \;\; n = 2, \cdots, N \tag{4}$$

Expression (2) is derived directly from property **P1**, i.e., the fact that when $L = 1$ the optimal service level is equal to the maximum traffic demand. Expression (3) is due to properties **P2** and **P3** which state that when the number of service levels is at least as large as the number of service requests, the amount of bandwidth is the same as that required by a continuous-rate network. The recursive expression (4) can be explained by noting that with request set $\mathcal{R}_n$ and $l+1$ service levels, the bandwidth $b_{l+1}$ of service level $l+1$ must be equal to the maximum traffic demand $d_n$. According to Lemma 3.1, the remaining $l$ optimal service levels must be assigned values from the traffic demands $d_i$ of the remaining $n-1$ service requests. In particular, the $l$-th service level can only take a value equal to one of the $n-l$ traffic demands $d_l, d_{l+1}, \cdots, d_{n-1}$. If the $l$-th level is assigned bandwidth equal to $d_q, q = l, \cdots, n-1$, the amount of total bandwidth needed is given by the expression in brackets in the right-hand side of expression (4). Taking the minimum over all possible values of $q$ provides the optimal value $C^\star(\mathcal{R}_n, l+1)$. The overall complexity of computing $C^\star(\mathcal{R}_N, L) = C^\star(\mathcal{R}, L)$ with the dynamic programming algorithm is $O(LN^2)$.

### 3.3. Results

We first demonstrate the operation of the dynamic programming algorithm by considering two small sets of service requests. For the results we present here we assume that $\lambda$-channels operate at a rate of 1 Gbps and that the minimum and maximum demands are equal to 100 Mbps and 600 Mbps, respectively. The first set consists of $N = 11$ traffic demands uniformly distributed in the interval [100,600], as shown in the first row of Table 1. The table also lists the optimal set of service levels for $L = 1, \cdots, 11$, as well as the corresponding value of $C^\star(\mathcal{R}, L)$.

While the example of Table 1 is a small one, it helps illustrate both the operation of the dynamic programming algorithm and the effects of the quantization process. First, we observe that the optimal

Table 1
Optimal service levels and corresponding $C^\star(\mathcal{R}, L)$ values for the first 11-request set $\mathcal{R}$

| Requests in Set $\mathcal{R}$ (Mbps) | | 100 | 150 | 200 | 250 | 300 | 350 | 400 | 450 | 500 | 550 | 600 | $C^\star(\mathcal{R}, L)$ (Mbps) |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Optimal Service Levels (Mbps) | $L = 1$ | | | | | | | | | | | 600 | 6600 |
| | $L = 2$ | | | | | 300 | | | | | | 600 | 5100 |
| | $L = 3$ | | | 200 | | | | 400 | | | | 600 | 4600 |
| | $L = 4$ | | 150 | | | 300 | | | 450 | | | 600 | 4350 |
| | $L = 5$ | | 150 | | 250 | | 350 | | 450 | | | 600 | 4200 |
| | $L = 6$ | 100 | | 200 | | 300 | | 400 | | 500 | | 600 | 4100 |
| | $L = 7$ | 100 | 150 | 200 | | 300 | | 400 | | 500 | | 600 | 4050 |
| | $L = 8$ | 100 | 150 | 200 | 250 | 300 | | 400 | | 500 | | 600 | 4000 |
| | $L = 9$ | 100 | 150 | 200 | 250 | 300 | 350 | 400 | | 500 | | 600 | 3950 |
| | $L = 10$ | 100 | 150 | 200 | 250 | 300 | 350 | 400 | 450 | 500 | | 600 | 3900 |
| | $L = 11$ | 100 | 150 | 200 | 250 | 300 | 350 | 400 | 450 | 500 | 550 | 600 | 3850 |

Table 2
Optimal service levels and corresponding $C^\star(\mathcal{R}, L)$ values for the second 11-request set $\mathcal{R}$

| Requests in Set $\mathcal{R}$ (Mbps) | | 93 | 97 | 141 | 146 | 148 | 430 | 442 | 445 | 448 | 572 | 589 | $C^\star(\mathcal{R}, L)$ (Mbps) |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Optimal Service Levels (Mbps) | $L = 1$ | | | | | | | | | | | 589 | 6479 |
| | $L = 2$ | | | | | 148 | | | | | | 589 | 4274 |
| | $L = 3$ | | | | | 148 | | | | 448 | | 589 | 3710 |
| | $L = 4$ | | 97 | | | 148 | | | | 448 | | 589 | 3608 |
| | $L = 5$ | | 97 | | | 148 | 430 | | | 448 | | 589 | 3590 |
| | $L = 6$ | | 97 | | | 148 | 430 | | | 448 | 572 | 589 | 3573 |
| | $L = 7$ | | 97 | 141 | | 148 | 430 | | | 448 | 572 | 589 | 3566 |
| | $L = 8$ | | 97 | 141 | | 148 | 430 | 442 | | 448 | 572 | 589 | 3560 |
| | $L = 9$ | 93 | 97 | 141 | | 148 | 430 | 442 | | 448 | 572 | 589 | 3556 |
| | $L = 10$ | 93 | 97 | 141 | | 148 | 430 | 442 | 445 | 448 | 572 | 589 | 3553 |
| | $L = 11$ | 93 | 97 | 141 | 146 | 148 | 430 | 442 | 445 | 448 | 572 | 589 | 3551 |

strategy appears to be to distribute the optimal service levels evenly along the interval [100,600]. This behavior is expected given that traffic demands are uniformly distributed. We note, however, that uniformly distributing the service levels along the range between minimum and maximum demand is *not* in general an optimal strategy, as the next example will demonstrate. We also clearly observe the law of diminishing returns in action: there is a sharp drop in the value of $C^\star(\mathcal{R}, L)$ when we add the second service level, but each additional level decreases the value of $C^\star(\mathcal{R}, L)$ by an even smaller amount. In particular, when $L = 5$ the amount of bandwidth required by the quantized network (4,200 Mbps) is already within 10% of the amount of bandwidth (3,850 Mbps) required by a continuous-rate network.

The second set of requests also consists of $N = 11$ requests in the same range as the first set, but now the traffic demands (shown in the first row of Table 2) form four clusters near the values of 100 Mbps, 150 Mbps, 450 Mbps, and 600 Mbps; these values correspond to the rate of Fast Ethernet and (roughly) the payload rates of OC-3, OC-9, and OC-12 channels, respectively. From Table 2 we observe that the optimal strategy is quite different than for the first set of requests. Specifically, for values of $L$ up to four, the optimal strategy is to assign the service levels to the demand with the highest value within a cluster; for instance, for $L = 2$ the optimal levels are 148 (the highest value among 141, 146, and 148 of the second cluster) and 589 (the highest value among 572 and 589 of the fourth cluster). We also see that with $L = 4$ optimal service levels, the quantized network requires less than 2% additional bandwidth compared to a continuous-rate network. Clearly, it makes little sense for a network with this set of traffic demands to provide more than 4 service levels. Another observation is that the optimal strategy is such that the optimal set of $l$ service levels, $l = 2, \cdots, L$, is always equal to the optimal set for $l-1$ plus an additional service level. Again, however, the strategy of incrementally adding service levels to a set of existing ones is *not*, in general, optimal; for a counterexample, refer to the optimal service level sets for $L = 3$ and $L = 4$ in Table 1.

We now study the effects of quantization in terms of the bandwidth penalty by considering a wide range of traffic demands. For the results presented here, we consider $\lambda$-channels operating at 10 Gbps, and we assume that service requests have bandwidth demands between 100-2,500 Mbps. We consider two different traffic distributions. Let $d$ be the bandwidth demand of a service request. The first distribution is a uniform one in the range [100,2500], i.e., $Pr[d = x] = 1/2400, x \in [100, 2500]$. The second distribution is a 4-cluster distribution similar to the one considered in the previous subsection, and such that traffic demands are clustered around the values 150 Mbps, 600 Mbps, 1,000 Mbps, and 2,450 Mbps:

$$Pr[d = x] = \begin{cases} \frac{0.2}{100}, & x \in [100, 200] \text{ or } x \in [550, 650] \text{ or } x \in [1000, 1100] \text{ or } x \in [2400, 2500] \\ \frac{0.2}{2000}, & x \in (200, 550) \cup (650, 1000) \cup (1100, 2400) \end{cases} \quad (5)$$

For each distribution, we considered request sets $\mathcal{R}$ of size $N = 100, 500, 1000$. For each value of $N$, we generated 30 different sets of requests drawn from the given distribution. For each set, we computed the $L$ optimal service levels, $L = 1, 2, \cdots, N$, using the dynamic programming algorithm. Figures 1-6 plot the "normalized bandwidth," defined as the ratio $C^\star(\mathcal{R}, L)/D(\mathcal{R})$ for the different request sets $\mathcal{R}$ and for several values of $L$. Since $D(\mathcal{R})$ is the bandwidth required by a continuous-rate network, the normalized bandwidth captures the performance penalty due to quantization.

Figures 1, 2, and 3 plot the normalized bandwidth for request sets of size equal to $N$=100, 500, and 1000, respectively, with traffic demands within each set drawn from the uniform distribution. We present results only for a number of service levels equal to $L$ =2,3,4,5,10, and $N$. Recall that $C^\star(\mathcal{R}, N) = D(\mathcal{R})$, therefore the normalized bandwidth for $L = N$ is equal to 1, as shown in the figures. For any $L < N$, $C^\star(\mathcal{R}, L)/D(\mathcal{R}) > 1$ when all $N$ requests in the set $\mathcal{R}$ are distinct. The plots for the different values of $L$ reinforce our previous observations. We first observe the diminishing returns of having one additional service level. We also see that with $L = 5$ service levels the bandwidth penalty is less than 20% of the bandwidth required by a continuous-rate network, while with $L = 10$ the penalty is less than 10%. These results are valid across the three request set sizes, as well as across all sets of a certain size. The natural conclusion is that a service provider has little to gain by offering more than $L = 10$ service levels.

Figures 4, 5, and 6 plot the normalized bandwidth for request sets of size equal to $N = 100, 500$, and 1000, respectively, but the traffic demands of these sets are drawn from the 4-cluster distribution in (5). The overall behavior of the plots for the various values of the set size $N$ and the number $L$ of service levels is very similar to that seen in Figures 1-3. The main difference is that the performance penalty due to quantization is lower for this 4-cluster distribution than for the uniform distribution. In other words, for a given value of $N$ and $L$, the curves for the 4-cluster distribution are lower than the curves for the uniform distribution. In particular, Figures 4-6 indicate that with $L = 5$ service levels the bandwidth penalty is about 10% of the bandwidth required by a continuous-rate network, and with $L = 10$ levels the penalty is about 5%. This result can be explained by the fact that the quantization penalty is strongly related to the entropy of the density function describing the probability distribution from which the requests are drawn. Since the uniform distribution has higher entropy than the 4-cluster distribution, the corresponding quantization penalty is also higher.

## 4. The Stochastic Optimization Problem

In practice, it is unlikely that the set of traffic demands will be known in advance, or that providers will dimension their networks for a fixed set of requests. Therefore, we need a method for computing the optimal set of service levels when only the distribution of the size of traffic requests is known. Such a distribution may, for instance, be obtained empirically. In the following, we extend the dynamic programming approach of the previous section to address this issue.

Let $f(x)$ be the probability density function of the size of service requests, i.e., $\int_0^{d_{max}} f(x)dx = 1$, where $d_{max} \le C$ denotes the maximum traffic demand and $C$ represents the capacity of a $\lambda$-channel. Let $\mathcal{R}$ be a set of $N$ traffic demands drawn from this distribution. Then, the expected total bandwidth

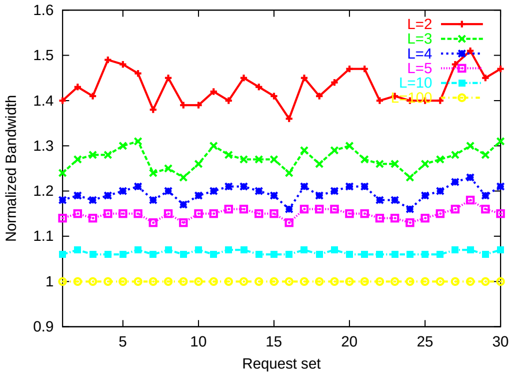


Figure 1. Sets of $N = 100$ requests uniformly distributed in [100,2500] (Mbps)

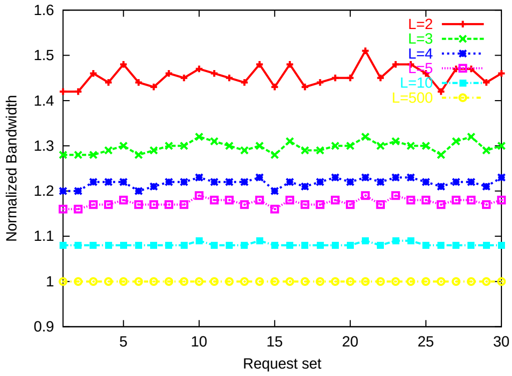


Figure 2. Sets of $N = 500$ requests uniformly distributed in [100,2500] (Mbps)

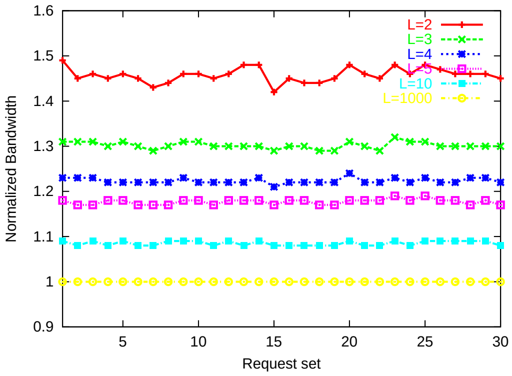


Figure 3. Sets of $N = 1000$ requests uniformly distributed in [100,2500] (Mbps)

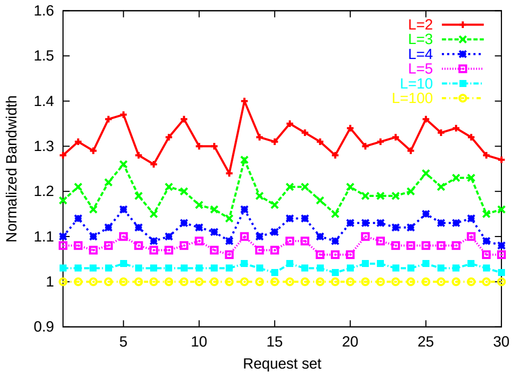


Figure 4. Sets of $N = 100$ requests from a 4-cluster distribution in [100,2500] (Mbps)

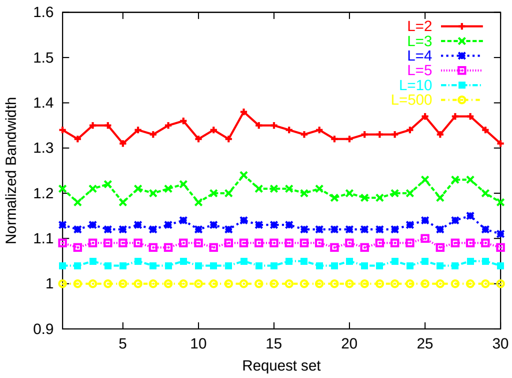


Figure 5. Sets of $N = 500$ requests from a 4-cluster distribution in [100,2500] (Mbps)

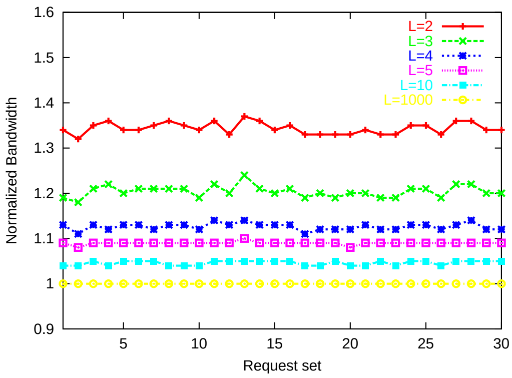


Figure 6. Sets of $N = 1000$ requests from a 4-cluster distribution in [100,2500] (Mbps)

$D(\mathcal{R})$ required by a continuous rate network to support all $N$ traffic demands is given by $D(\mathcal{R}) = NE[d]$, where $E[d]$ is the expected bandwidth demand of a service request, $E[d] = \int_0^{d_{max}} xf(x)dx$.

Consider now a quantized network offering a set $\mathcal{S}_L$ of $L$ service levels. We let $C(\mathcal{R}, \mathcal{S}_L)$ denote the expected bandwidth needed to support all $N$ requests in set $\mathcal{R}$. For the given probability density function $f(x)$, we have that $C(\mathcal{R}, \mathcal{S}_L) = \sum_{l=1}^{L} b_l \int_{b_{l-1}}^{b_l} Nf(x)dx$, where the integral represents the expected number of service requests with a traffic demand between $b_{l-1}$ and $b_l$. The expected penalty due to quantization is $C(\mathcal{R}, \mathcal{S}_L) - D(\mathcal{R})$.

As in the previous section, our objective is to minimize $C(\mathcal{R}, \mathcal{S}_L)$. One difficulty that arises in this case in terms of applying a dynamic programming technique, is that service levels may take any value in the continuous interval $(0, d_{max}]$. To overcome this difficulty, we constrain the service levels to a discrete set of values uniformly distributed along the interval $(0, d_{max}]$. Specifically, for a given integer $K > 1$, let $\Delta_K = d_{max}/K$. The $L$ service levels are allowed to take values only from the set $\{k\Delta_K, k = 1, \cdots, K\}$. Since $\lim_{K\to\infty} \Delta_K = 0$, the set of service levels obtained using the dynamic programming algorithm described next will approach the optimal set for large values of $K$.

Let $C^\star(\mathcal{R}, L)$ denote the expected total bandwidth required to carry all $N$ requests in $\mathcal{R}$ under an optimal set $\mathcal{S}_L$ of $L$ service levels. Let $\mathcal{R}_k$ denote the set of service requests with demand no more than $k\Delta_K$ (note that $\mathcal{R}_K = \mathcal{R}$): $\mathcal{R}_k = \{r_i \in \mathcal{R} : d_i \le k\Delta_K\}, k = 1, \cdots, K$. We can now compute $C^\star(\mathcal{R}, L)$ recursively as follows:

$$C^\star(\mathcal{R}_k, 1) = k\Delta_K \int_0^{k\Delta_K} Nf(x)dx, \;\; k = 1, \cdots, K \tag{6}$$

$$C^\star(\mathcal{R}_1, l) = \Delta_K \int_0^{\Delta_K} Nf(x)dx, \;\; l = 1, \cdots, L \tag{7}$$

$$C^\star(\mathcal{R}_k, l+1) = \min_{q=l,\cdots,k-1} \left\{ C^\star(\mathcal{R}_q, l) + k\Delta_K \int_{q\Delta_K}^{k\Delta_K} Nf(x)dx \right\} \tag{8}$$

Expressions (6)-(8) are slightly different than the corresponding expressions (2)-(4) in that the recursion is not in terms of the number $n$ of service requests but in terms of the number $k$ of possible values for the service levels. For instance, expression (6) states that with set $\mathcal{R}_k$ and $l = 1$, the single service level must be equal to $k\Delta_K$ in order to accommodate the request with the highest demand in $\mathcal{R}_k$. Expressions (7) and (8) can be explained in a similar manner.

The complexity of the dynamic programming algorithm is $O(LK^2)$. While the set of $L$ service levels we obtain depends on the value of $K$, results to be presented shortly indicate that the algorithm converges to the optimal set quickly as the value of $K$ increases. Also, while the final value of $C^\star(\mathcal{R}, L)$ scales with $N$, the actual value of $N$ does not affect the actual set of optimal service levels, and solving (6)-(8) with $N = 1$ is sufficient to obtain the optimal set of service levels.

### 4.1. Results

We now study the quantization penalty and the effect of the value of $K$ in expressions (6)-(8) on the optimality of the service levels computed by the dynamic programming algorithm. We again assume that $\lambda$-channels operate at 10 Gbps and that traffic demands take values in the range between 0 and 2,500 Mbps. We consider six different traffic distributions, shown in Figure 7. The first is a uniform distribution, while the next three are of a cluster type, whereby traffic demands are concentrated in two, three, or four clusters, respectively, in the range [0,2500]. The last two distributions are such that the density function is linearly increasing or decreasing, respectively, in the range [0,2500].

For each traffic distribution we used the dynamic programming algorithm of the previous subsection to obtain the $L$ optimal service levels. We used $N = 1$ in the three expressions (6)-(8) since as we mentioned earlier, the actual value of $N$ does not affect optimality. To study how the value of $K$ in (6)-(8) affects the solution, we run the dynamic programming algorithm for $K = 1, \cdots, 1000$. For each value of $K$, we obtained the $L$ optimal service levels for $L = 1, \cdots, K$.

Figures 8-13 plot the expected normalized bandwidth $C^\star(\mathcal{R}, L)/D(\mathcal{R})$ of a quantized network with $L$ optimal service levels, against the value of $K$. Each of the six figures corresponds to one of the six traffic distributions shown in Figure 7. Note that for $L = K = 1000$ we essentially have a continuous-rate

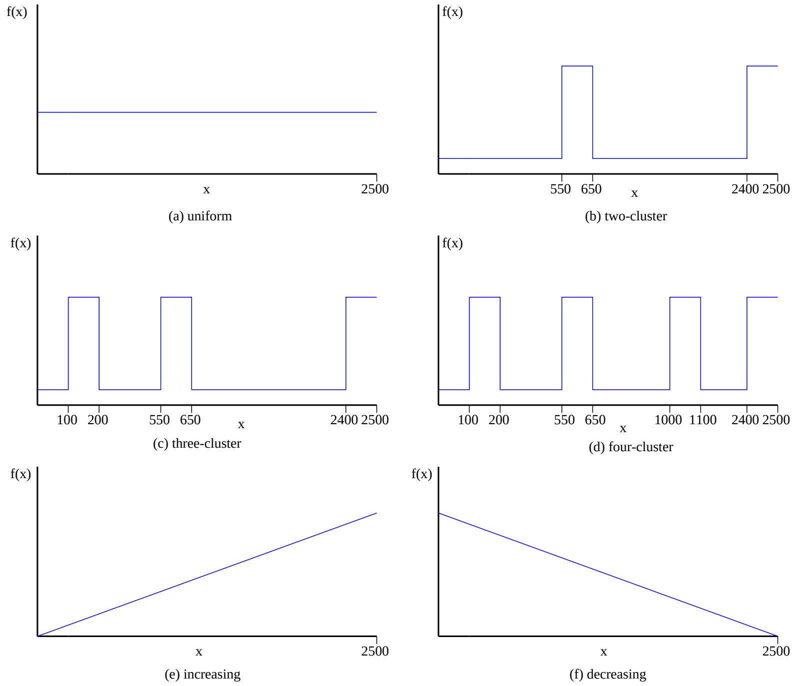


Figure 7. Probability density function $f(x)$ for the six traffic distributions (not in scale)

network; this is borne out in the figures since the normalized bandwidth for these values of $L$ and $K$ is equal to one for all density functions.

There are two important observations we can make from the six figures. The first is that for a given value of $L$, the normalized bandwidth levels off quickly as $K$ increases. This behavior implies that a relatively small value of $K$ (e.g., 100) is sufficient to obtain the optimal service levels. The second observation is that the effect of the number $L$ of service levels on the quantization penalty is similar to that observed for discrete request sets. In particular, with $L = 10$ service levels, a quantized network uses about 10% more bandwidth than a continuous-rate network. Overall, our results indicate that a continuous-rate network offers little advantage over a quantized network with at least 10 service levels.

## 5. Conclusions

We have presented a dynamic programming algorithm for computing a set of optimal service levels for optical networks in which traffic streams are aggregated for transport over a direct $\lambda$-channel. Our results can greatly simplify the process of specifying a service level agreement by allowing network providers to offer an optimized menu of services. We are currently extending our work to the case where traffic streams are described by a vector of descriptors, for instance, bandwidth *and* maximum burst size. In this case the benefits of quantization are even greater since the space of potential levels scales with the product of the possible values for each traffic descriptor.

## REFERENCES

1. P. Ashwood-Smith *et al.* Generalized MPLS – signaling functional description. IETF Draft <draft-ietf-mpls-generalized-signaling-06.txt>, April 2001. Work in progress.
2. E. Rosen *et al.* Multiprotocol label switching architecture. RFC 3031, January 2001.

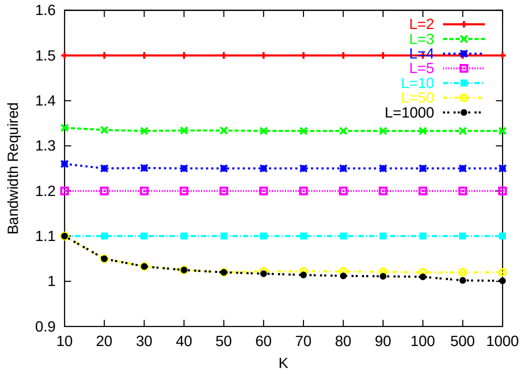


Figure 8. Optimal bandwidth $C^\star(\mathcal{R}, L)$, uniform distribution

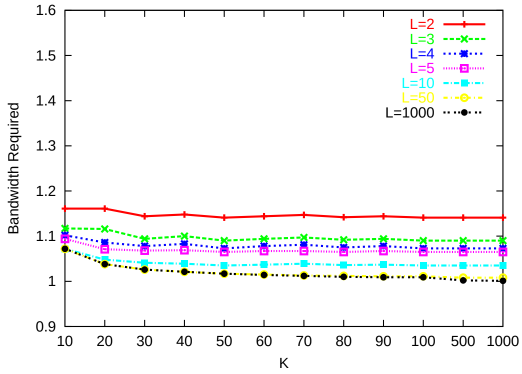


Figure 9. Optimal bandwidth $C^\star(\mathcal{R}, L)$, 2-cluster distribution

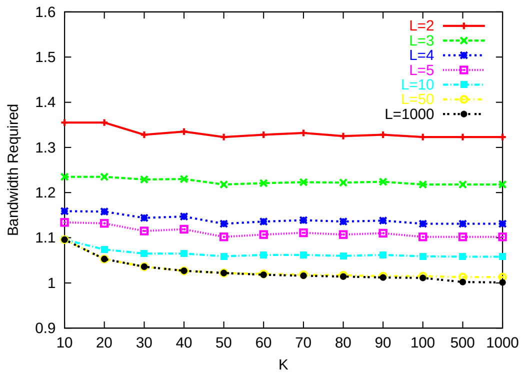


Figure 10. Optimal bandwidth $C^\star(\mathcal{R}, L)$, 3-cluster distribution

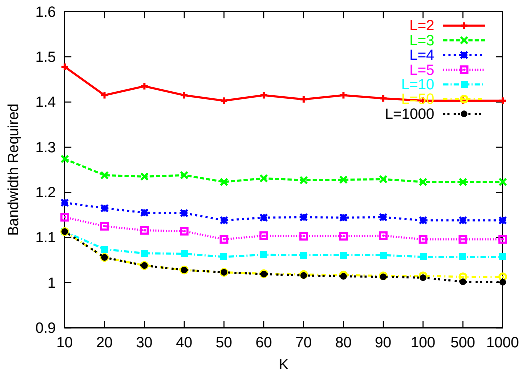


Figure 11. Optimal bandwidth $C^\star(\mathcal{R}, L)$, 4-cluster distribution

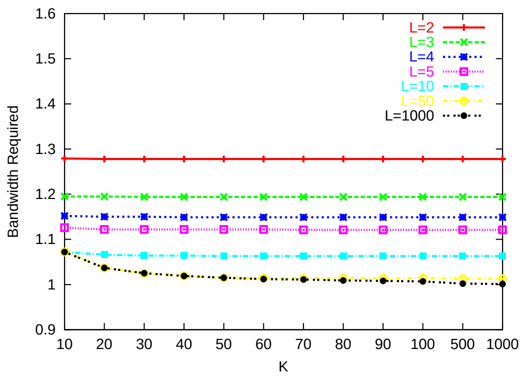


Figure 12. Optimal bandwidth $C^\star(\mathcal{R}, L)$, increasing distribution

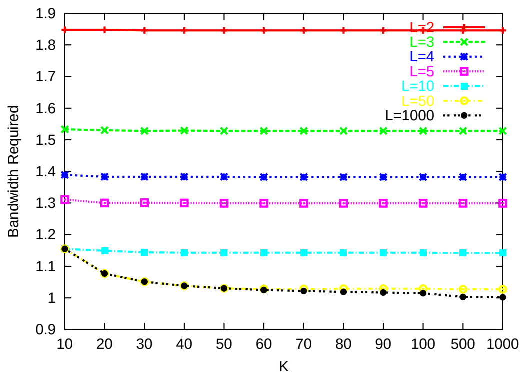


Figure 13. Optimal bandwidth $C^\star(\mathcal{R}, L)$, decreasing distribution