RWA in WDM Rings: Efficient Exact Formulations Based on Maximal Independent Sets

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Outline

- Routing and Wavelength Assignment (RWA)
- Existing ILP Formulations
- New ILP Formulations Based on
  - MIS Decomposition
  - MIS Selection
- Numerical Results
- Conclusion and Future Research Directions

Why “RWA in Rings”?

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- Why “RWA”?
  - subproblem of all optical network design problems
    $\longrightarrow$ speed up “what-if” analysis to test sensitivity of solution to forecast demands, cost projections, price structures, etc.
  - intellectually appealing!

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  - intellectually appealing!
- Why “Rings”?
  - ring topologies prevalent today and in foreseeable future
  - insight into RWA problem in mesh topologies

Routing and Wavelength Assignment (RWA)

- Fundamental control problem in optical networks
- Objective: for each connection request determine a lightpath, i.e.,
  - a path through the network, and
  - a wavelength
- Two variants:
  1. online RWA: connection requests arrive/depart dynamically
  2. static RWA: a set of traffic demands to be established simultaneously

Static RWA

- Input:
  - network topology graph $G = (V, E)$
  - traffic demand matrix $T = [t_{sd}]$
- Objective:
  - minRWA: establish all demands with the minimum # of λs
  - maxRWA: maximize established demands for a given # of λs
- Constraints:
  - wavelength continuity: each lightpath uses the same λ along path
  - distinct wavelength: lightpaths using the same link assigned distinct λs
- NP-hard problem (both variants)

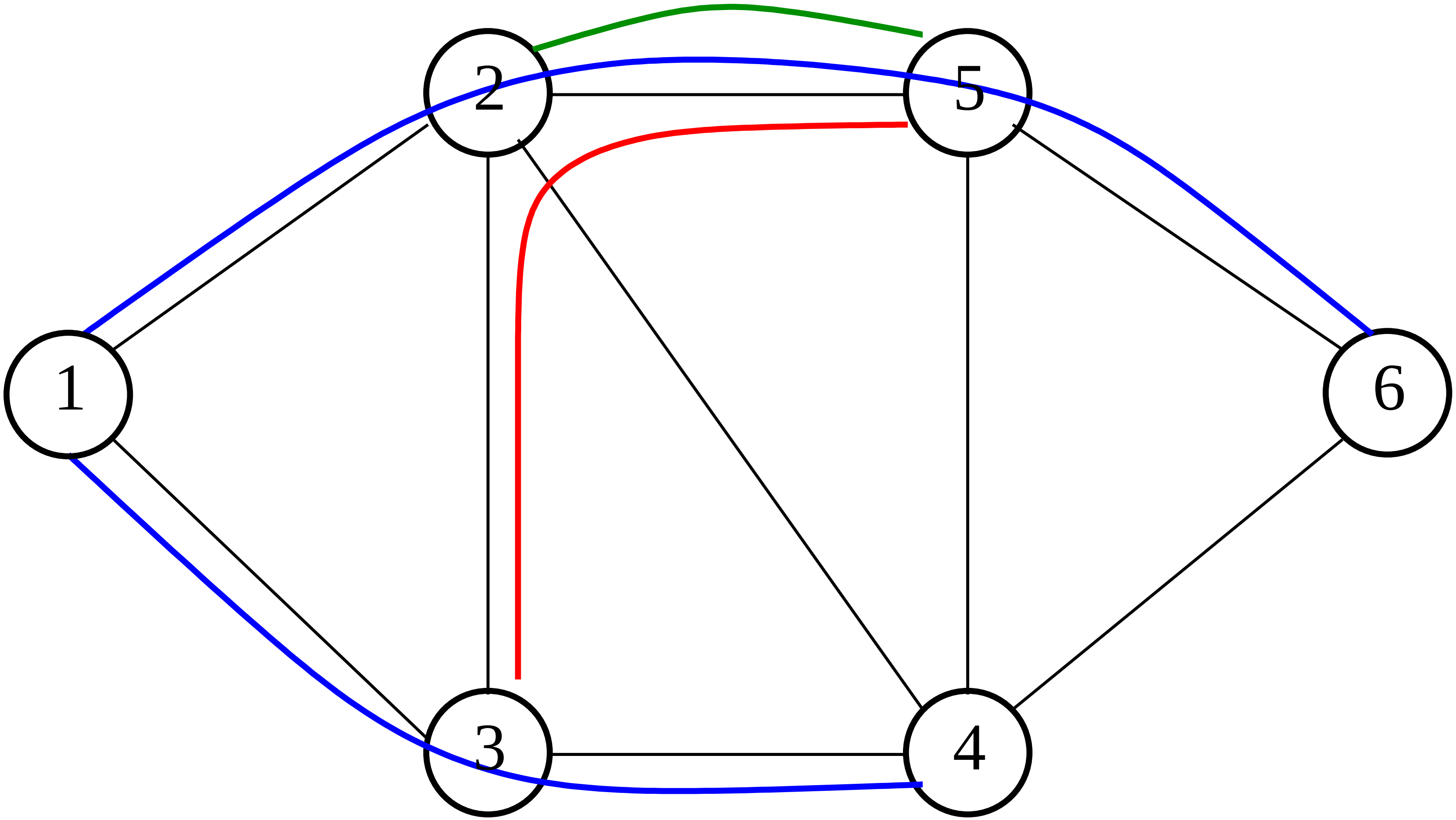
Solution Approaches

1. ILP formulations
   - Link-based
   - Path-based
   - MIS-based
2. Heuristics
   - Decomposition: R & WA
   - Multi-layer graph
   - . . .

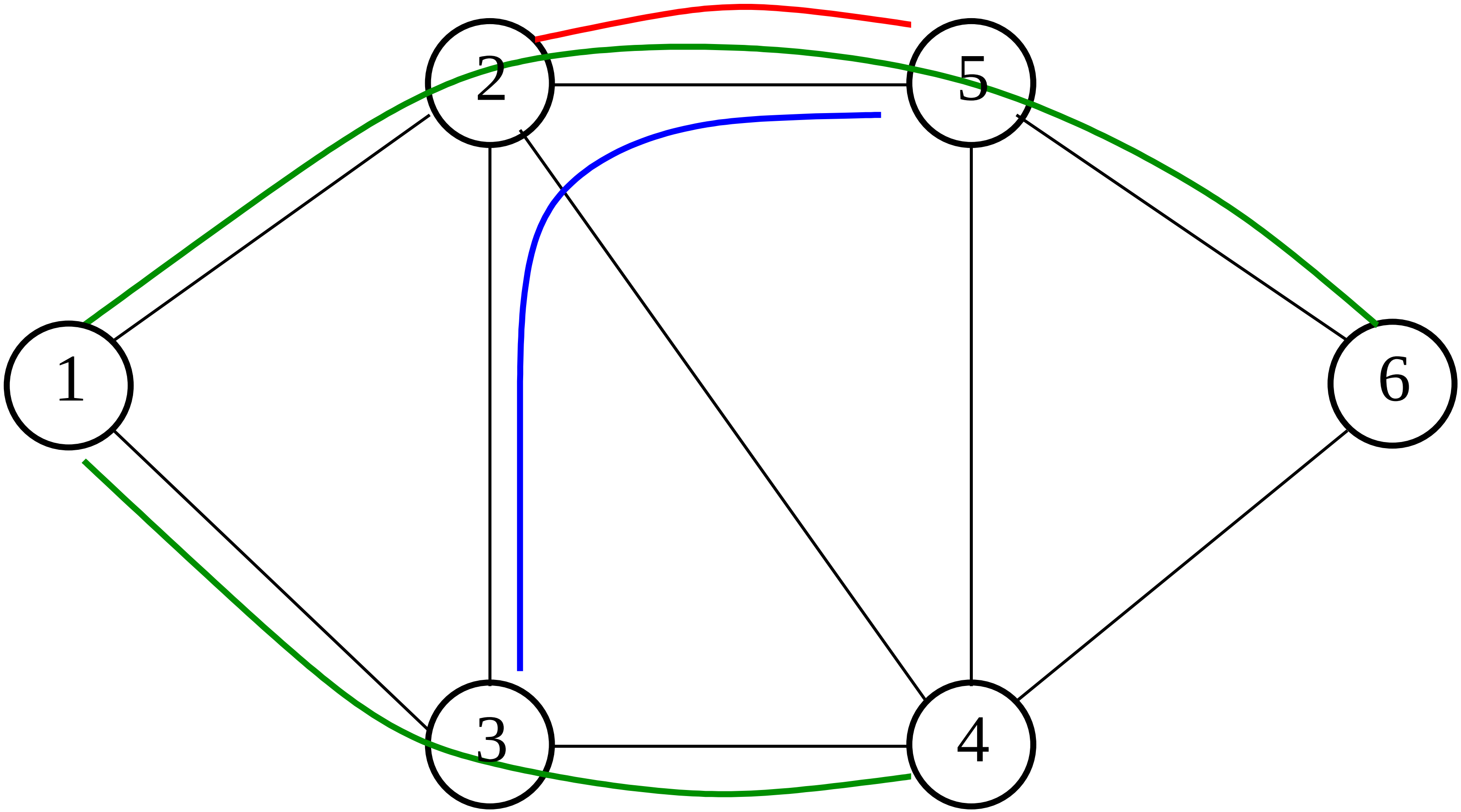
Challenges

- Existing approaches do not scale well with:
  - network size
  - number of wavelengths
- Quality of heuristics is difficult to characterize
- Large λ regime not explored

RWA Example

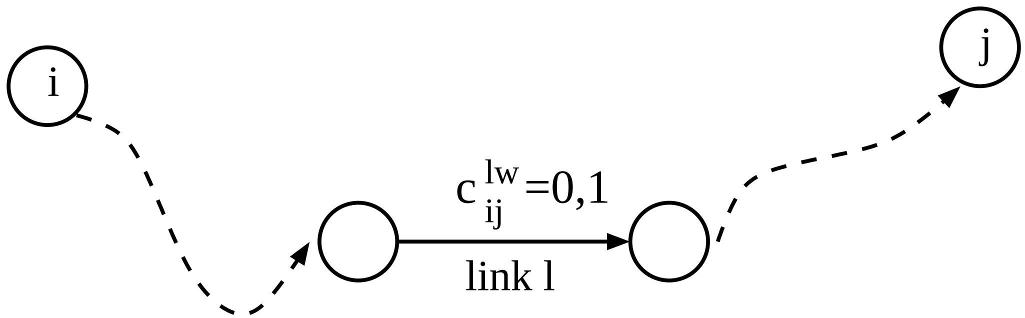


RWA: Symmetry



Link ILP Formulation

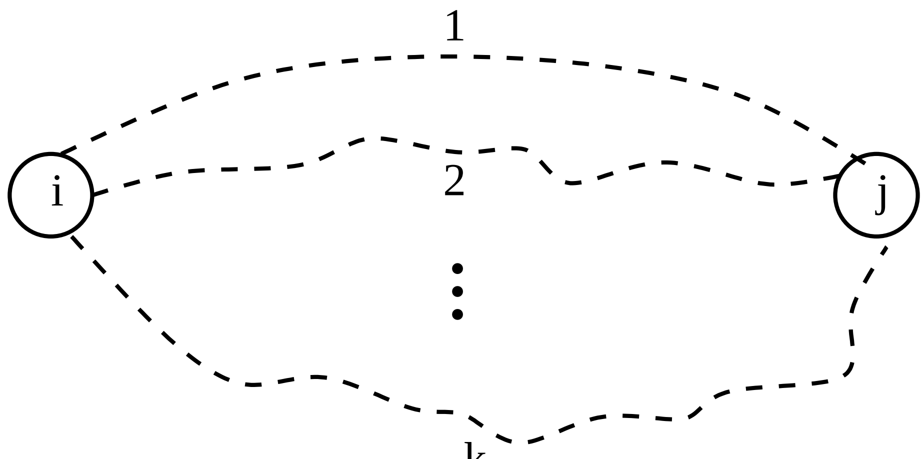
- Nodes/links are entities of interest
- Focus on traffic demand to and from nodes, on links



- Bridging variable: demand between nodes on links

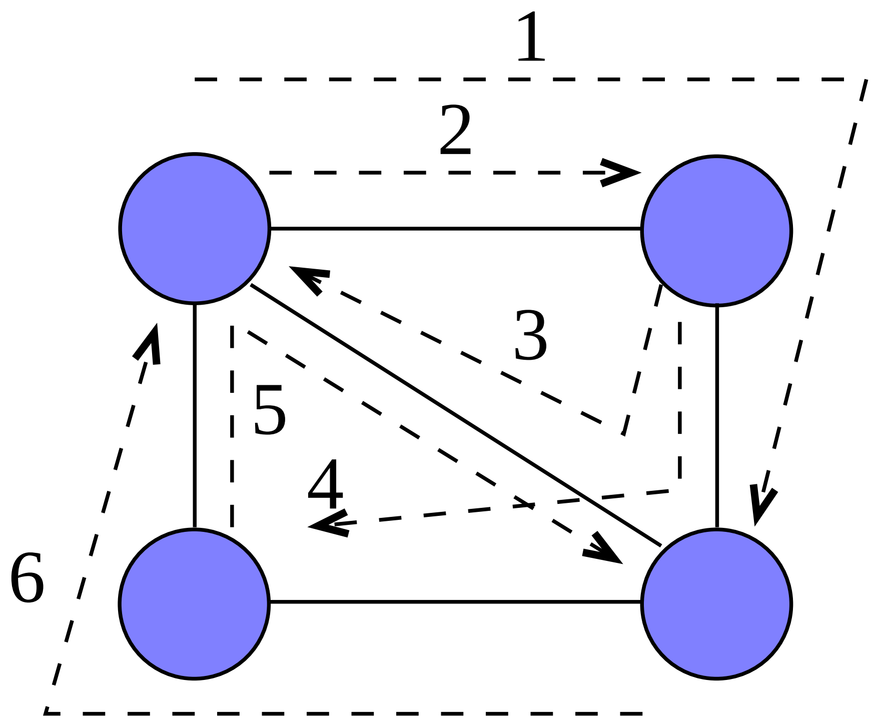
Path ILP Formulation

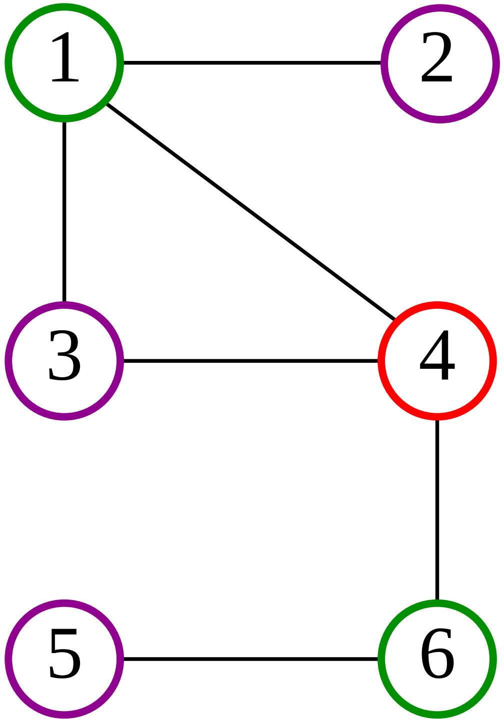
- Nodes/paths are entities of interest
- Demand is still between nodes
- For each given demand node pair, list all paths
  $\longrightarrow$ typically, a subset of all paths



- assign variable to path traffic flow $\longrightarrow$ implicitly identifies demand
- for each link, sum up path flow variables
  $\longrightarrow$ constrain with capacities

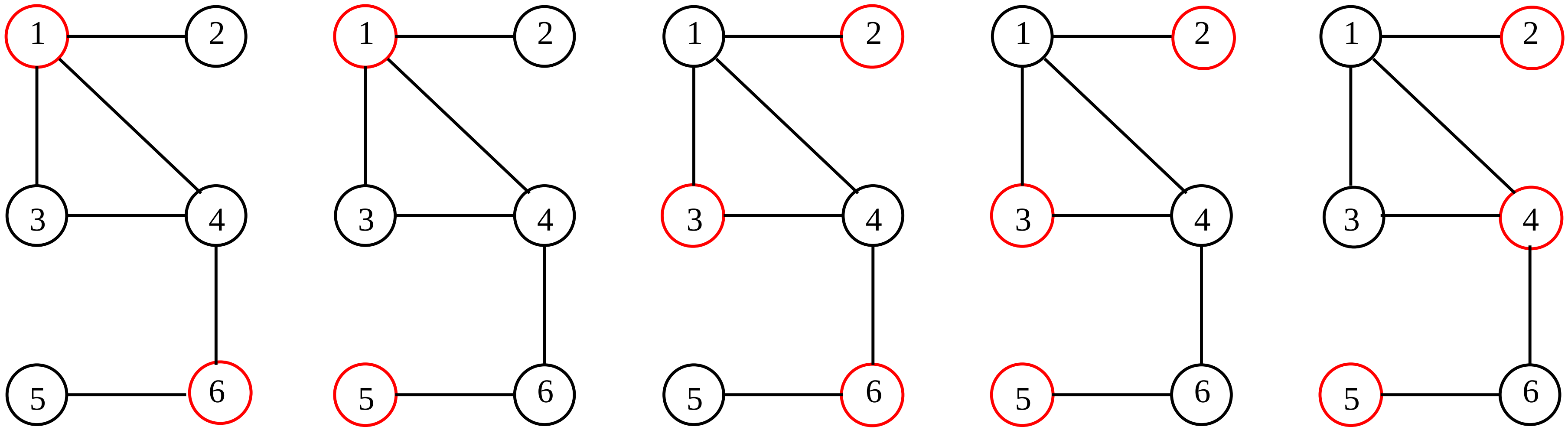
RWA As Graph Coloring





Maximal Independent Sets

- Independent set: a set of vertices in a graph no two of which are adjacent
- Maximal independent set: not a subset of any other independent set



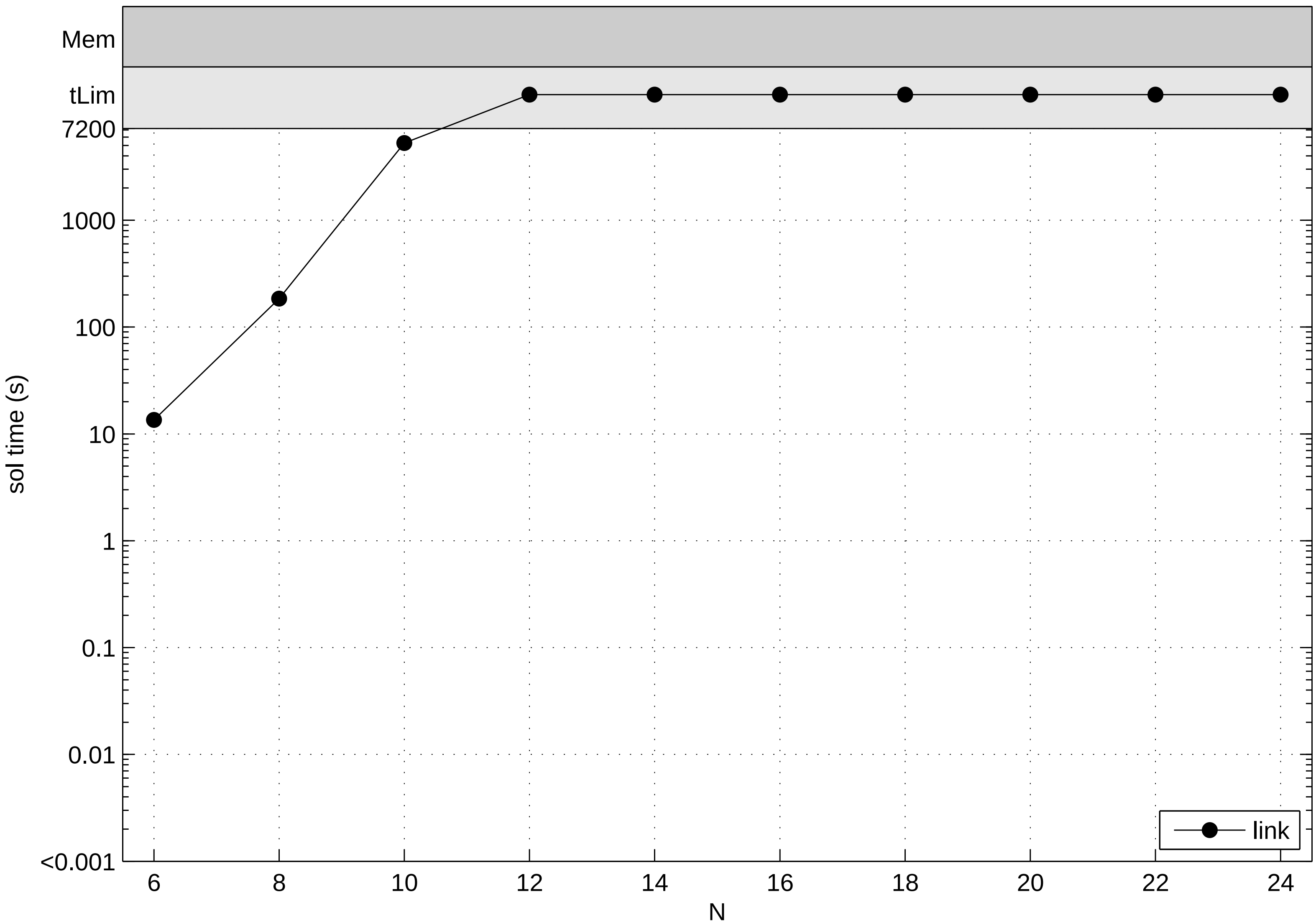
MIS ILP Formulation

- Precompute k paths for each source-destination pair
- Create the path graph G_p:
  - each node in G_p corresponds to a path in the original network
  - two nodes connected in G_p if corresponding paths share a link
- Enumerate the MISs of G_p
- Set up ILP to assign wavelengths to each MIS

Comparison

Formulation	# Variables	# Constraints	Symmetry?
Link	$O(N^4 W)$	$O(N^3 W)$	Yes
Path	$O(N^2 W)$	$O(N^2 W)$	Yes
MIS	$O(3^{N^2/3})$	$O(N^2)$	No $\longrightarrow$ future-proof

Running Time Results, $W = 120$



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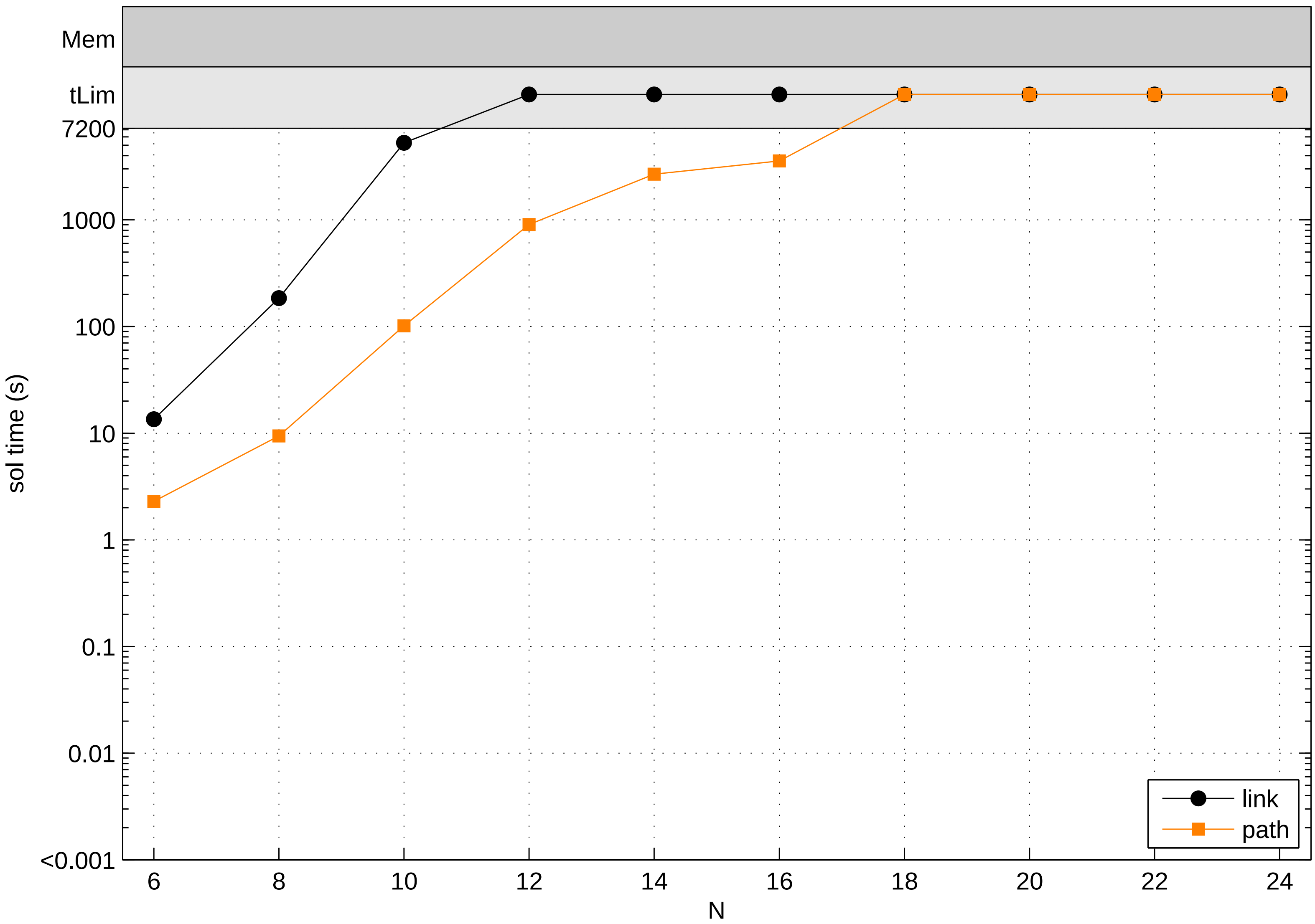

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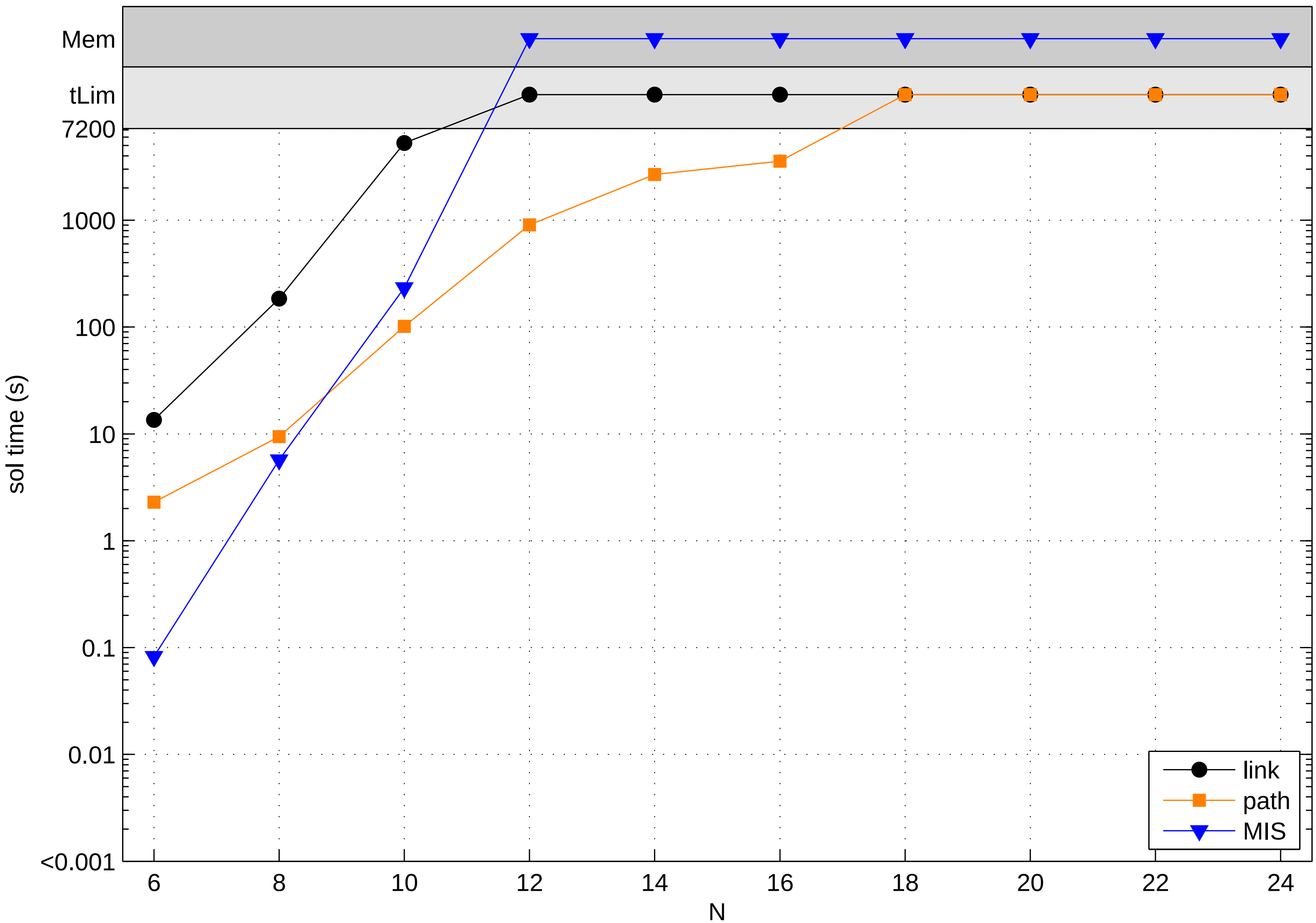
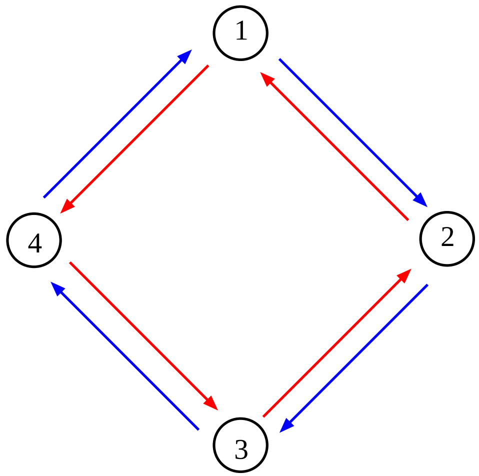


MIS Decomposition for Rings: MISD-2



- Clockwise paths do not intersect with counter-clockwise paths:

$$G_p \;=\; G_p^{cw} \cup G_p^{ccw}$$

- M, M^{cw}, M^{ccw} : # of MISs of G_p, G_p^{cw}, G_p^{ccw}:

$$M^{cw} \;=\; M^{ccw} \;=\; \sqrt{M}$$

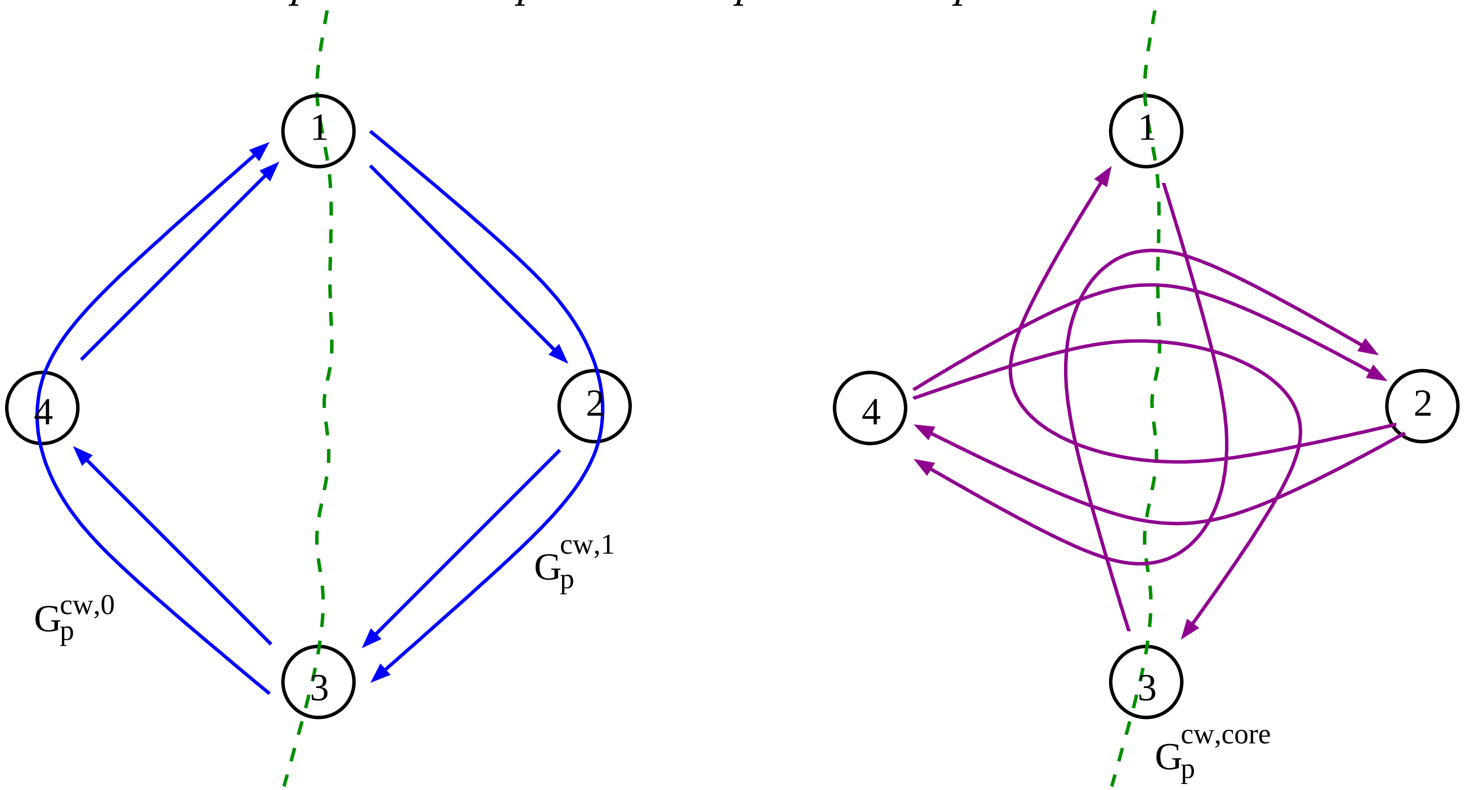
$\longrightarrow$ orders of magnitude decrease in # of variables/size of formulation

- Slight modifications to formulation

Further Decomposition: MISD-4

- Consider clockwise direction only
  $\longrightarrow$ similar steps for counter-clockwise
- Partition ring in two parts such that:

$$G_p^{cw} = G_p^{cw,0} \cup G_p^{cw,1} \cup G_p^{cw,core}$$



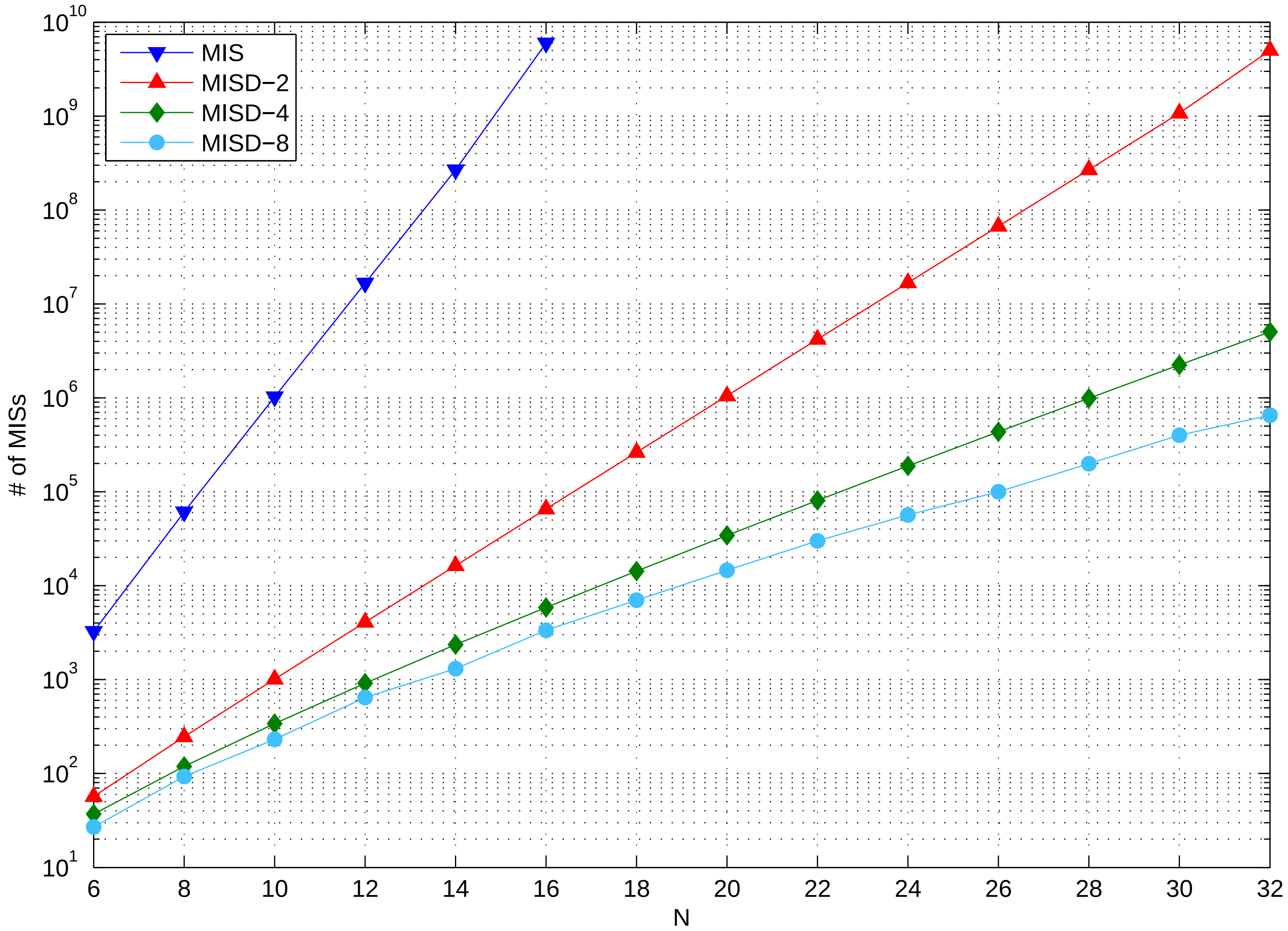
MISD-4 (cont'd)

- Express each MIS m of G_p^{cw} as:

$$m = m^0 \cup m^1 \cup q$$

- Modify the formulation appropriately
  - # MIS variables $\downarrow$
  - # constraints $\uparrow$
- Recursively partition the two ring parts to effect higher-order decompositions (MISD-8, MISD-16, . . .)

Results: # of MIS Variables



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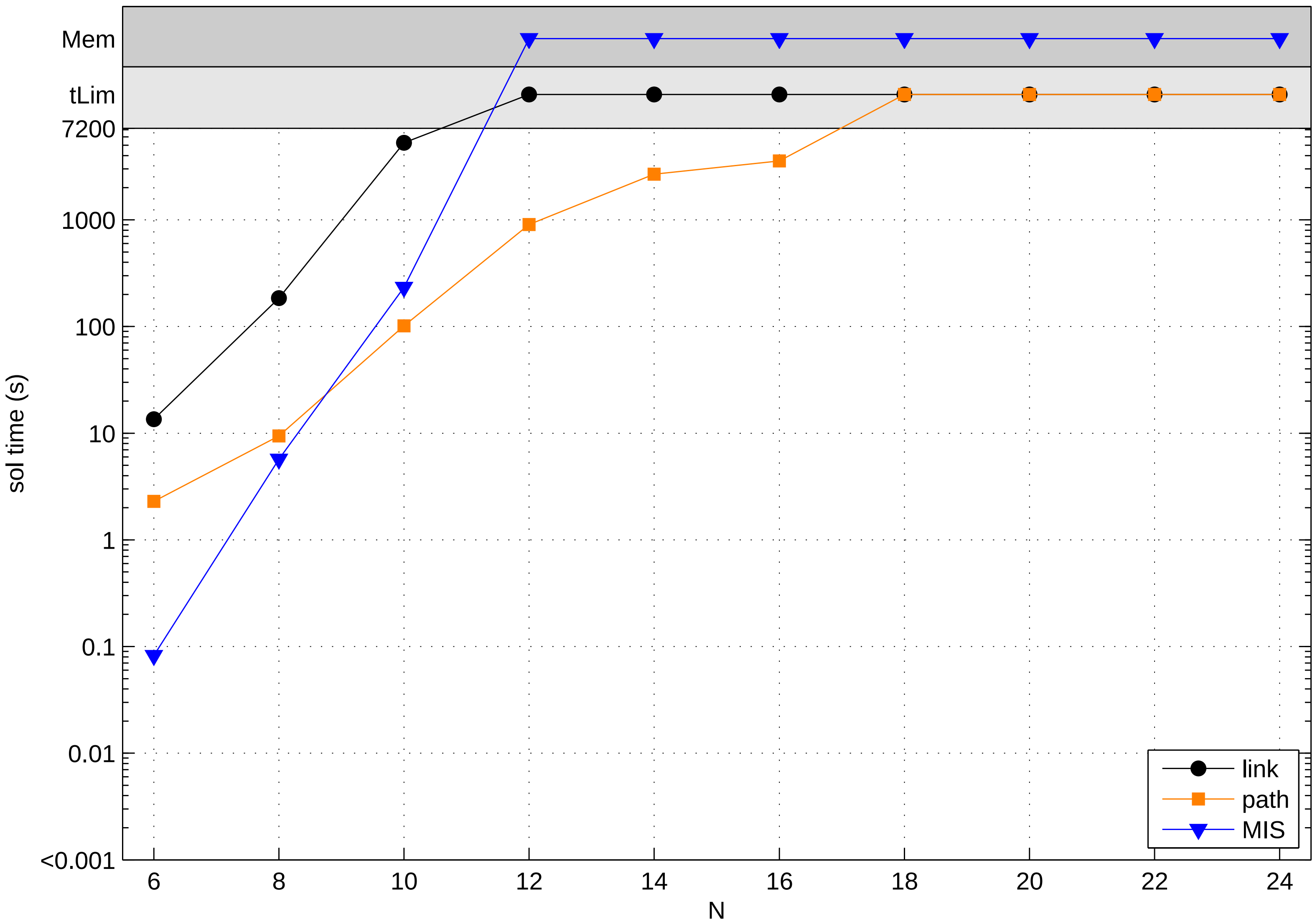

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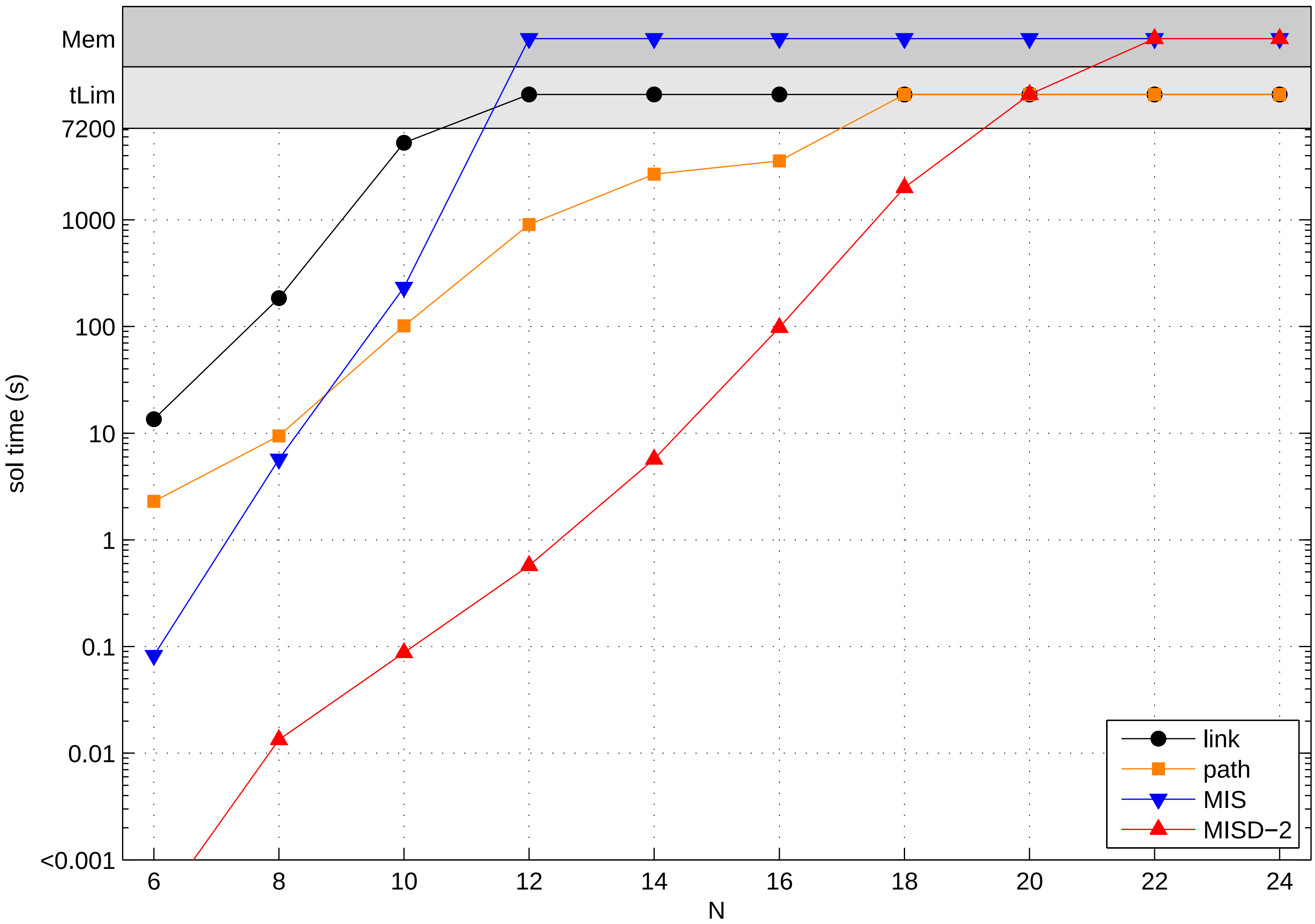

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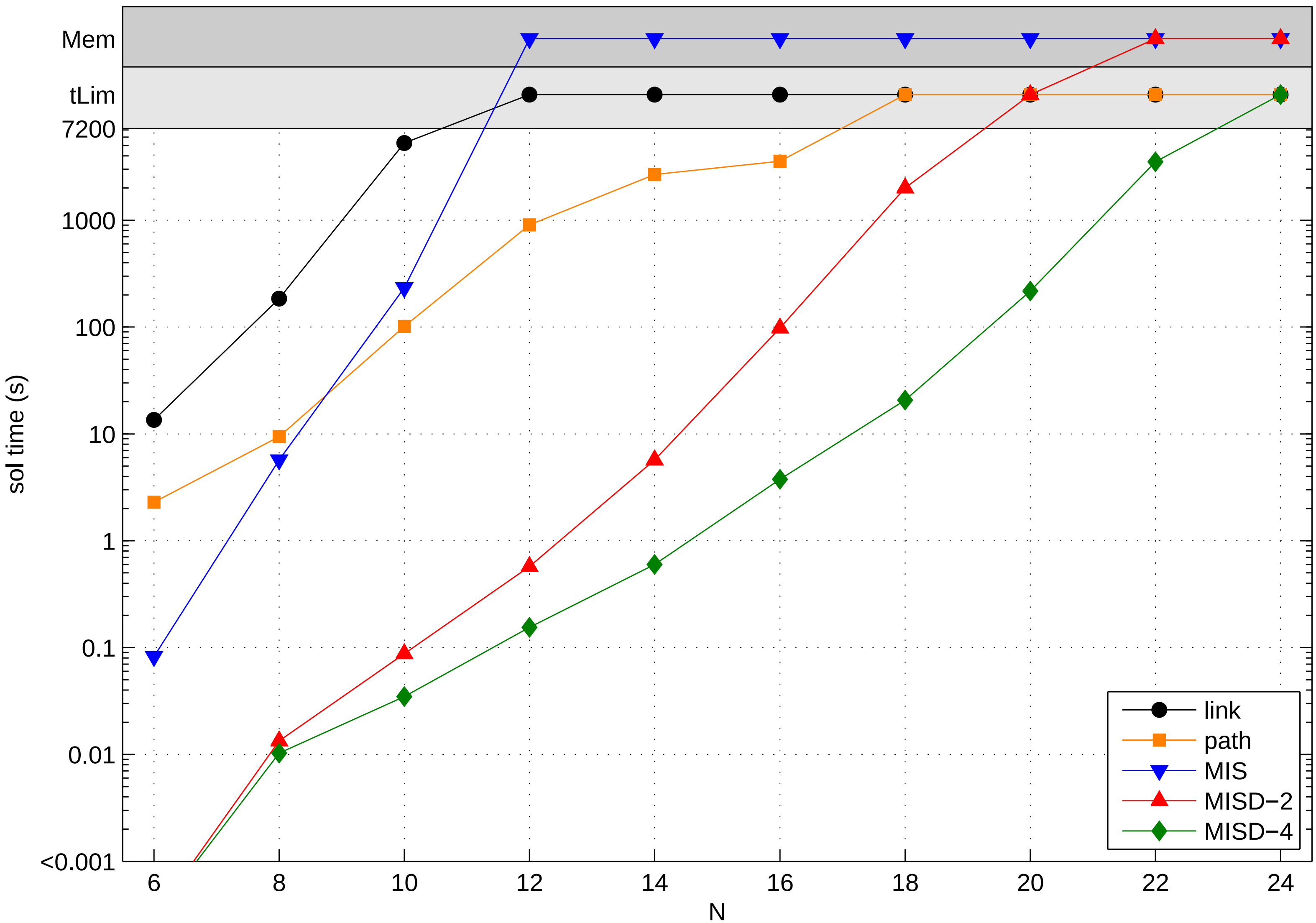

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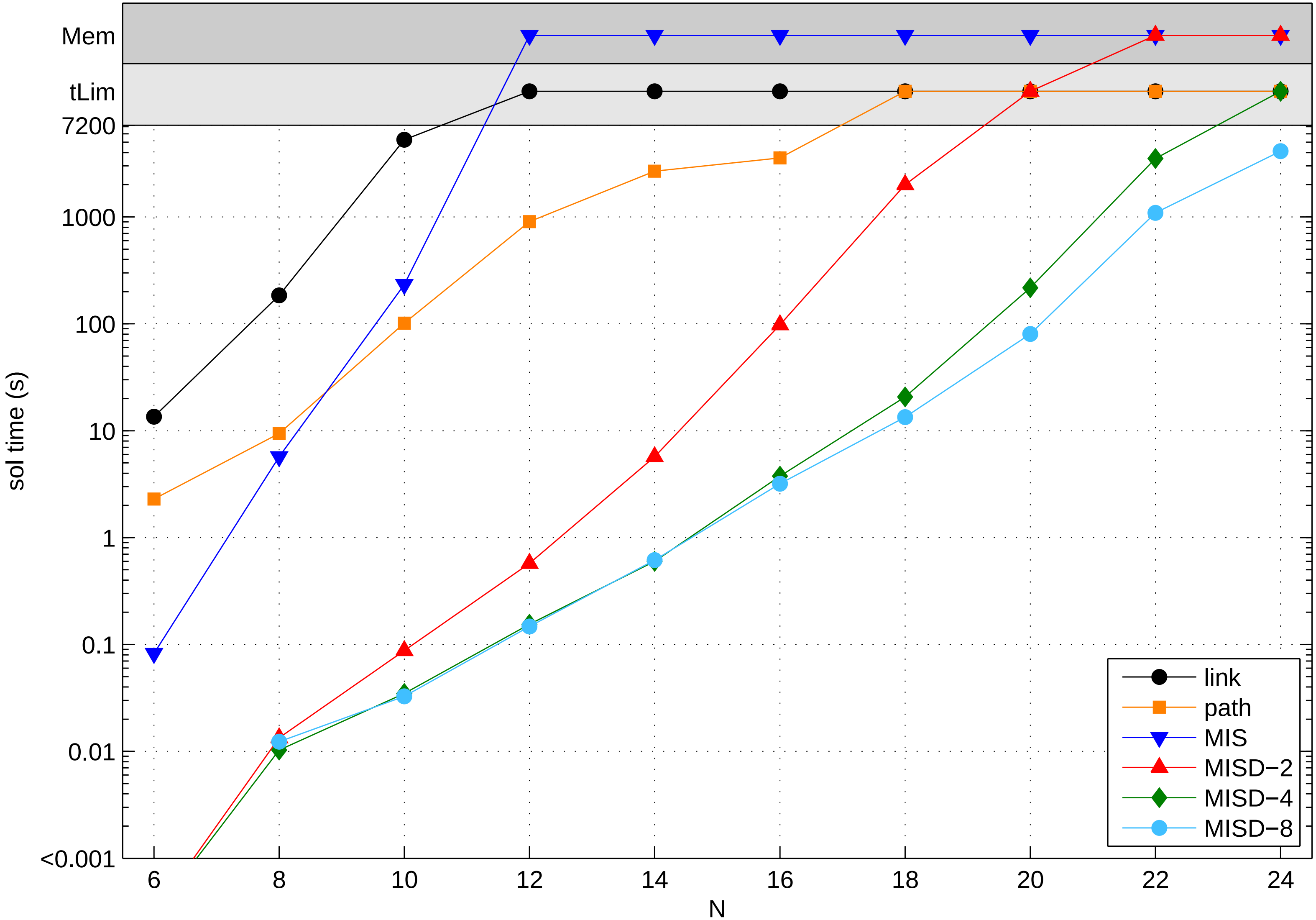
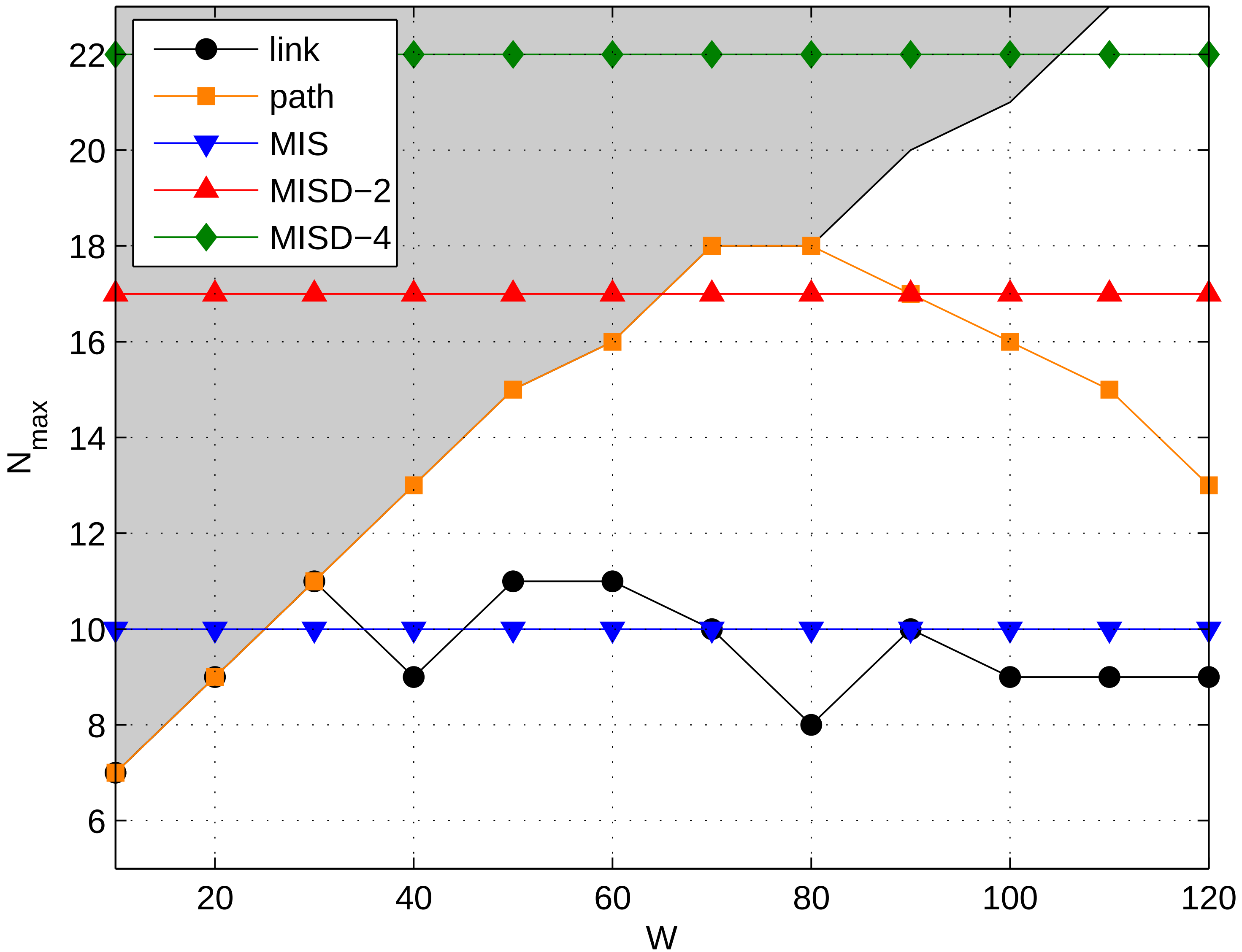


Results: Scalability with W



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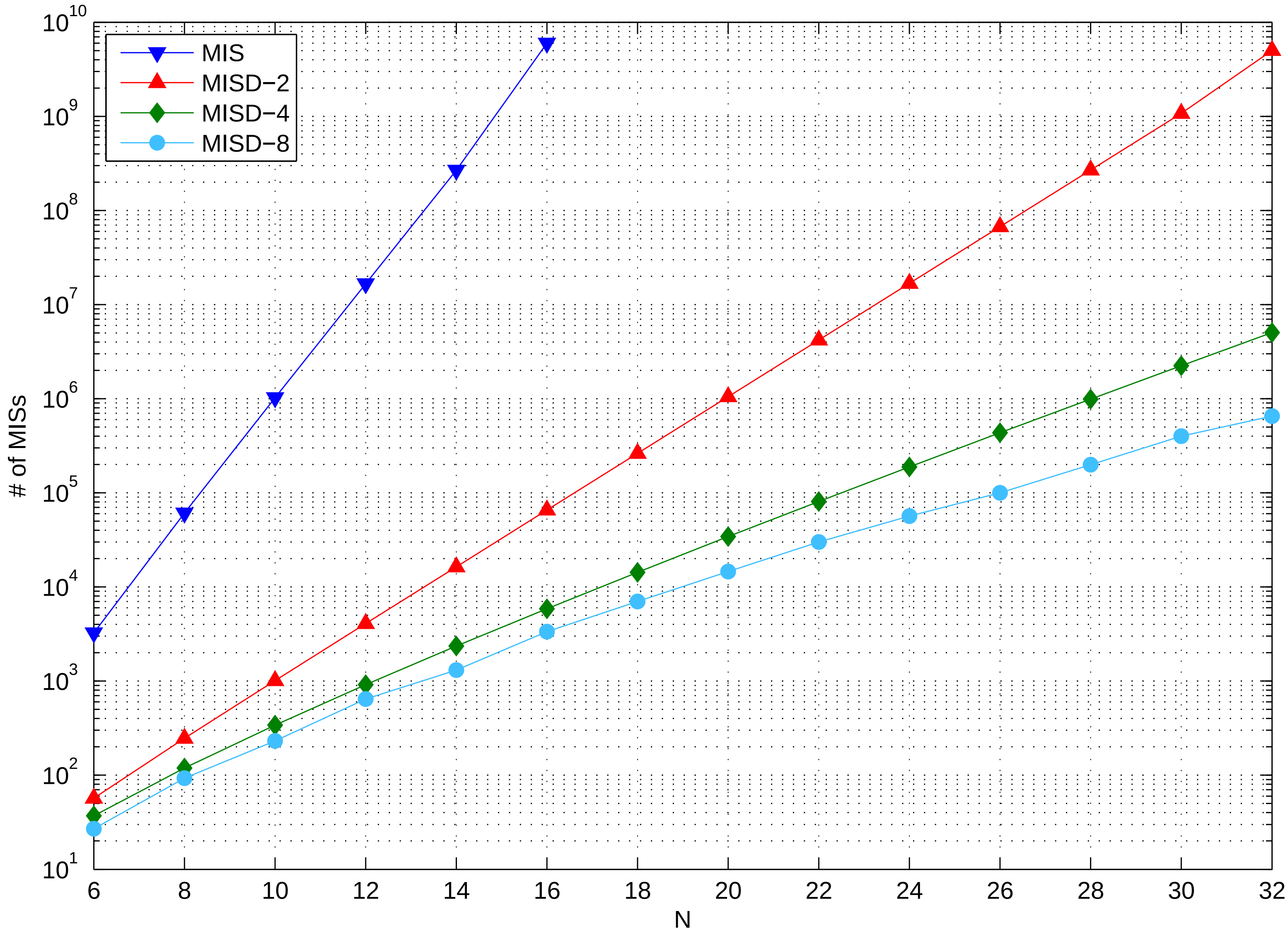
- Can we apply MIS decomposition to mesh networks?
  - yes – and it works well
  - but: size of initial MIS set orders of magnitude larger
    $\longrightarrow$ back to the drawing board

of MIS Variables



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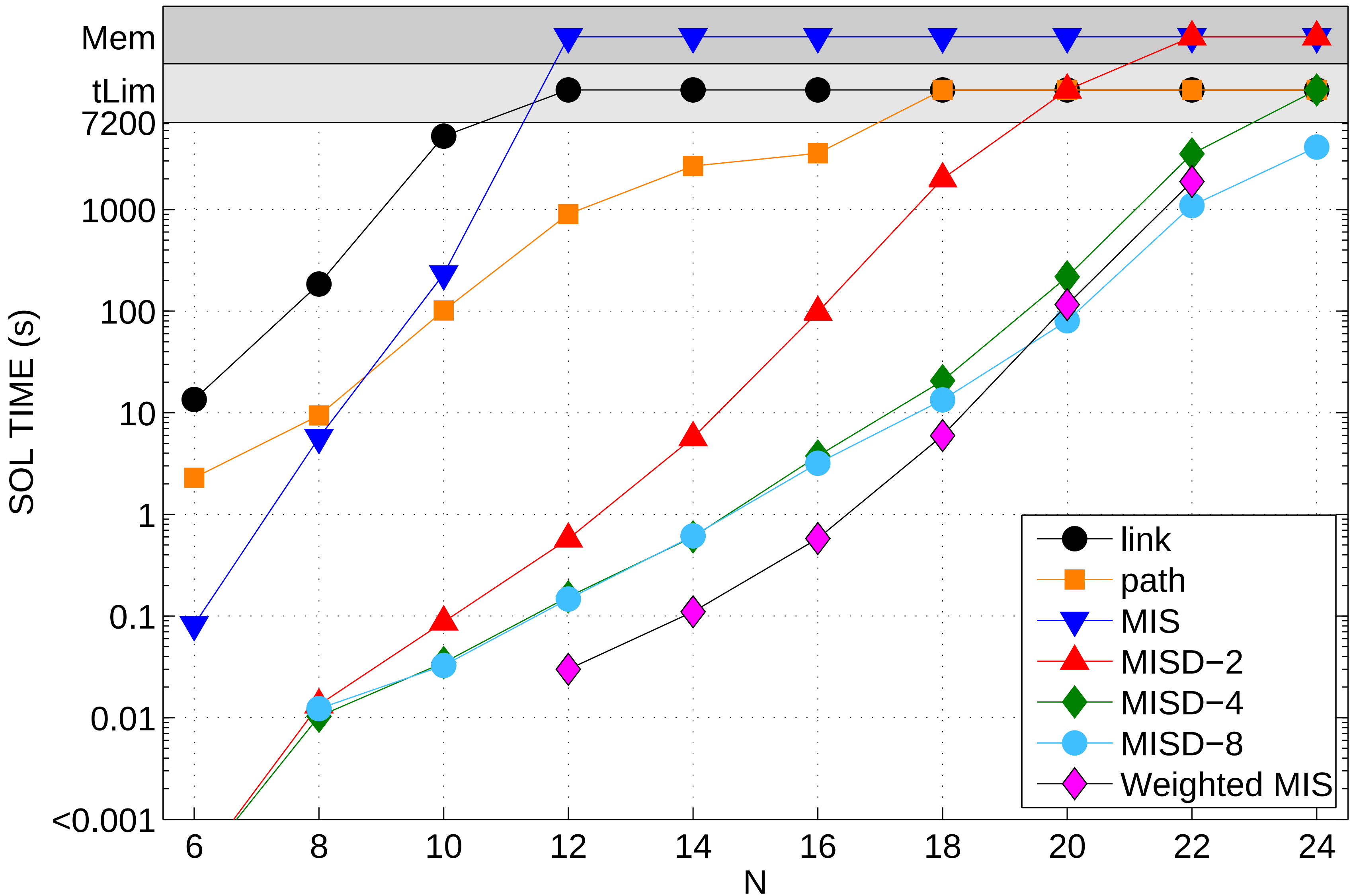

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  $\longrightarrow$ Some MIS variables important, others not
- Can we identify the important ones?

MIS Selection

- Prune useless MIS variables
  $\longrightarrow$ those containing paths with no traffic
- Rank remaining MIS variables in decreasing order of weight:
  - path (node) weight:

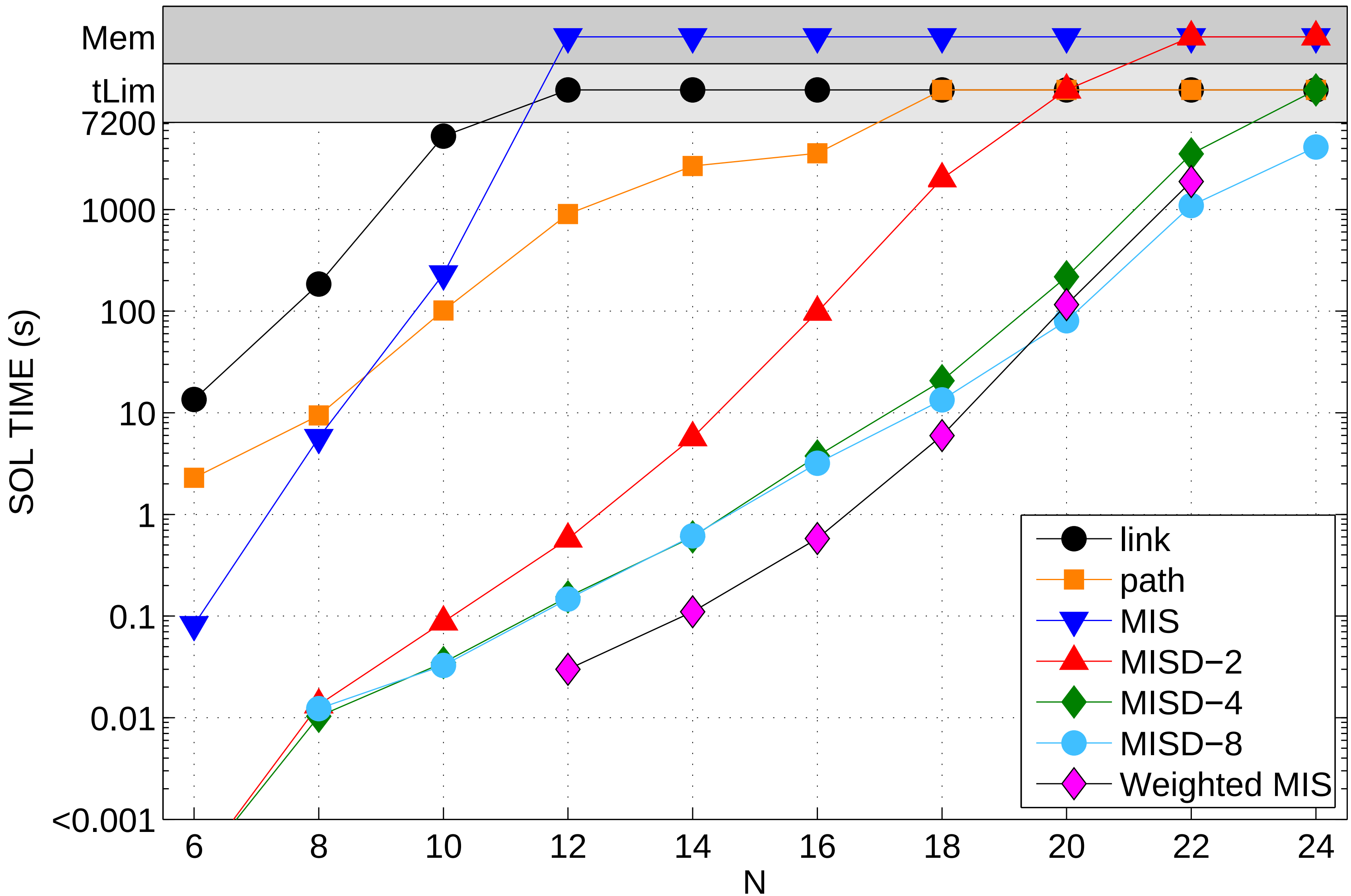
$$w = \text{degree}^2 \ \times \ \text{traffic}$$

  - MIS weight:

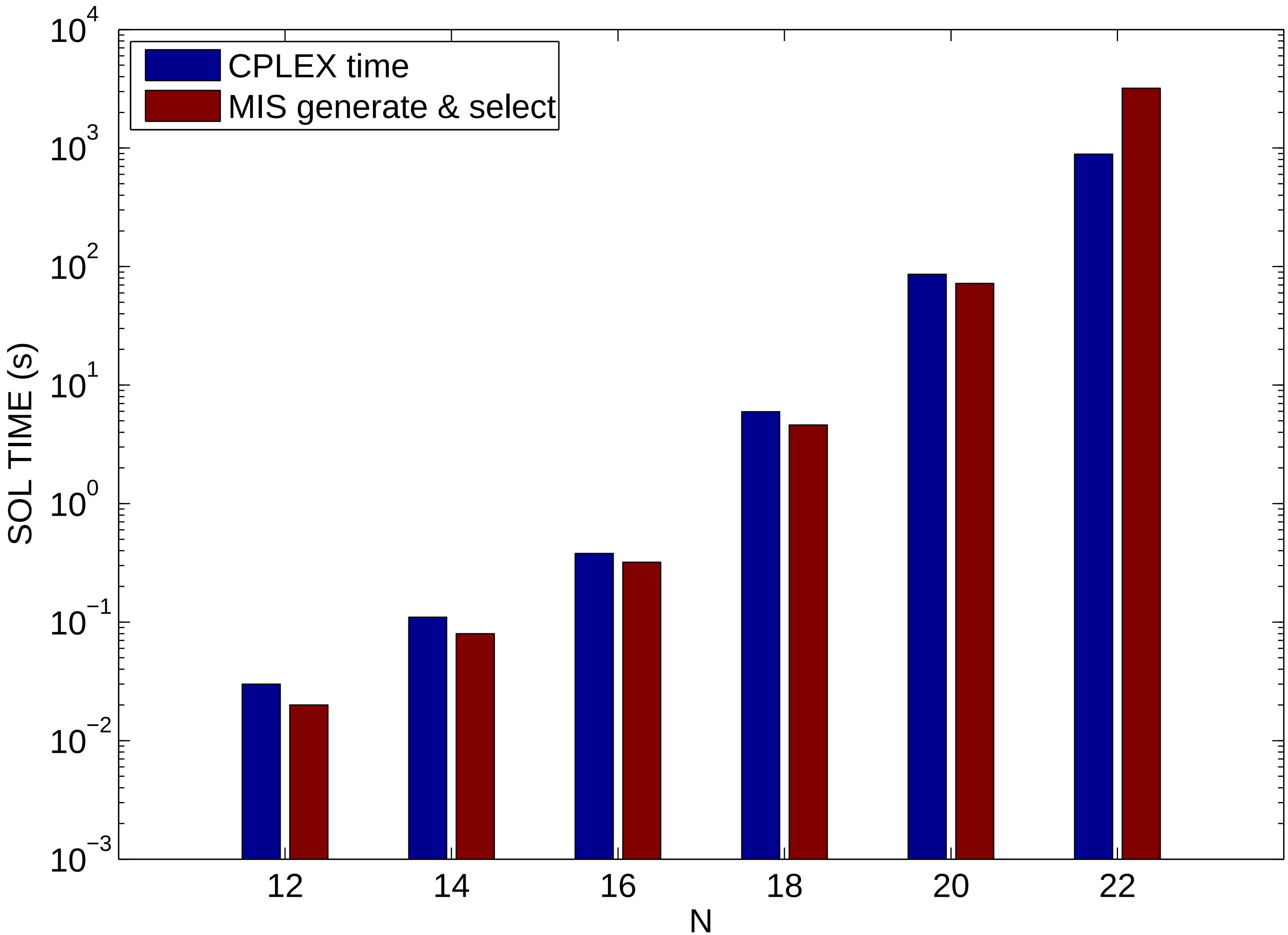
$$\sum_{\text{node } i \,\in\, \text{MIS}} w_i$$

- Include only top 10% of ordered MIS variables in formulation

Results


Mem
tLim
7200
1000
100
10
1
0.1
0.01
<0.001
SOL TIME (s)
6
8
10
12
14
16
18
20
22
24
N
link
path
MIS
MISD−2
MISD−4
MISD−8
Weighted MIS

Tradeoff


CPLEX time
MIS generate & select
SOL TIME (s)
10^{4}
10^{3}
10^{2}
10^{1}
10^{0}
10^{-1}
10^{-2}
10^{-3}
12
14
16
18
20
22
N

MIS Generation

- Large rings and mesh networks:
  - bottleneck shifts from CPLEX to enumeration of MIS variables
  - MIS set cannot fit in memory
- New algorithms needed: enumerate only most promising MIS variables
  - topic of ongoing research

Conclusion & Ongoing Research

- RWA problem can be solved efficiently in rings
  $\longrightarrow$ extensive “what-if” analysis now possible
- Current research focuses on:
  - extending MIS selection to mesh networks
  - efficient ILP formulations for optical network design problems
    - incorporate MIS decomposition for RWA
    - employ problem-specific knowledge