Scalable Traffic Grooming in Optical Networks

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Abstract: We develop a new solution approach for the traffic grooming problem by decomposing it into a virtual topology and traffic routing (VTTR) subproblem, and the RWA subproblem. We solve the VTTR subproblem with a new partial LP relaxation technique.

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1. Introduction

Traffic grooming is a fundamental design problem in optical networks to aggregate individual traffic requests onto wavelengths so as to improve bandwidth utilization across the network and minimize the use of network resources. Offline version of such network design problems have been shown to be NP-hard [1]. They have been formulated as integer linear programs (ILPs) and assume the existence of a traffic matrix representing the demands between node pairs. Basic ILP formulations of the problem are available in [2] and [3].

The basic ILP formulation and most variants of ILP formulations to solve traffic grooming problem suffer from two main challenges: scalability and wavelength fragmentation. We develop a new decomposition algorithm and partial LP relaxation technique for the traffic grooming problem, which addresses the above challenges well.

2. Basic ILP Formulation and Challenges

We are interested in minimizing the total number of lightpaths used in the network. Hence, we consider the following minimization problem that we refer to as *TG*.

Problem 2.1 (TG) *Given graph G, number of wavelengths W, wavelength capacity C, and traffic demand matrix T, establish the minimum number of lightpaths to carry all traffic demands.*

The ILP formulations to the above problem consist of an objective function of minimizing the number of lightpaths established, and three sets of constraints: virtual topology and traffic routing constraints, lightpath routing constraints, and wavelength assignment constraints (the actural formulations are omitted due to the space limit).

There are two essential challenges to the above formulations. The first one is that they are solvable only for small network topologies [4], due to the enormous number of constraints and variables. Another challenge is wavelength fragmentation. Since the objective is a function that depends only on the number of lightpaths and is independent of the number of wavelengths used to color the lightpaths, the ILP solver will not make any attempt to minimize the number of wavelengths. Such an approach will result in severe fragmentation of wavelengths.

3. A New Decomposition of Traffic Grooming

We decompose the *TG* problem defined earlier into two subproblems, the *virtual topology and traffic routing (VTTR)* subproblem, and the *routing and wavelength assignment (RWA)* subproblem.

3.1. Virtual Topology and Traffic Routing (VTTR)

Definition 3.1 (VTTR) *Given the number N of nodes in the graph G of TG, the wavelength capacity C, and traffic demand matrix T, establish the minimum number of lightpaths to carry all trafic demands.*

The ILP formulations to *VTTR* subproblem have the same objective function, but under only one of constraints, the virtual topology and traffic routing constraints.

Note that the *VTTR* problem does not take as input the network graph G, only the traffic demand matrix T (and, hence, the number of nodes, N). Consequently, the output of the problem is simply the set of lightpaths to be established but *not* the (physical) paths that these lightpaths take in the network. The virtual topology determined by *VTTR* will be reconciled with the physical topology using the second subproblem, as we discuss shortly.

3.2. *Routing and wavelength Assignment (RWA)*

Definition 3.2 (RWA) *Given the graph G of TG and the set of lightpath demands L determined by the solution to VTTR, route the lightpaths on the physical topology of G and assign a wavelength to each lightpath so as to minimize the number of distinct wavelengths required.*

The RWA problem is one of selecting a path and wavelength for each lightpath, subject to capacity and wavelength constraints. It is a fundamental problem in optical network design, and has been studied extensively. In [5] we developed an exact ILP formulation based on maximal independent sets (MIS) that solves the RWA problem in rings of size up to $N = 16$ nodes (the maximum size supported by SONET technology and hence *de facto* maximum size of deployed ring networks) in just 2-3 seconds, several orders of magnitude faster than earlier known solutions. We have also developed a path-based formulations that solve the RWA problem in mesh networks up to two orders of magnitude faster than existing techniques [6]. Therefore, we solve the RWA subproblem using the techniques in [5,6] rather than using the corresponding part of the formulation of the *TG* problem.

3.3. *Sequential Solution to the VTTR and RWA Problems*

We propose to solve the *TG* problem by sequentially solving its two subproblems: 1). Solve *VTTR* to obtain the set L of lightpaths to be established, and the routing of traffic demands over these lightpaths. 2). Solve *RWA* to find a path and wavelength for each lightpath in the set L so as to minimize the number of distinct wavelengths used in the solution.

Recall that the first step of the solution produces a set L of lightpaths that are determined only by the traffic demands and are not tied to the physical topology of the network. However, the second step routes the lightpaths over the physical links of the network, hence ensuring that the final solution is consistent with the network topology.

The following two lemmas state the properties of this sequential solution.

Lemma 3.1 *Let $P^\star_{TG}$ and $P^\star_{VTTR}$ denote the number of lightpaths returned by the optimal solutions to the TG and VTTR problems, respectively. Then: $P^\star_{VTTR} \;\leq\; P^\star_{TG}$.*

Proof. The *VTTR* subproblem is a relaxed version of the original *TG* problem with two sets of constraints removed. Hence, the objective value of an optimal solution to *VTTR* cannot be greater than that of an optimal solution to *TG*.

Lemma 3.2 *Let $W^\star_{RWA}$ be the number of wavelengths returned as the optimal solution to the RWA subproblem that takes as input the optimal solution $S^\star_{VTTR}$ of the VTTR subproblem. If $W^\star_{RWA} \leq W$, where W is the number of available wavelengths given as input to the original TG problem, then $S^\star_{VTTR}$, together with the lightpath routing and wavelength assignment determined by the RWA subproblem, is an optimal solution to TG.*

Proof. According to Lemma 3.1, the number of lightpaths in the solution $S^\star_{VTTR}$ is such that $P^\star_{VTTR} \leq P^\star_{TG}$. After the *RWA* is solved, the routing and wavelength assignment of the lightpaths in $S^\star_{VTTR}$ satisfy all the physical topology and wavelength assignment constraints. Hence, the final result of sequentially solving the two subproblems is also a feasible solution to the original problem *TG*, i.e., $P^\star_{VTTR} \geq P^\star_{TG}$, from which the result of the lemma follows.

The practical implication of Lemma 3.2 is that whenever the network is not wavelength (bandwidth) limited, the sequentially solving *VTTR* and *RWA* will yield an optimal solution to the original *TG* problem that also minimizes the number of wavelengths used for the given set of lightpaths.

4. Partial LP Relaxation of VTTR

We define the relaxed version of VTTR as the following:

Definition 4.1 *(VTTR-rlx) Given the number N of node in the graph G of TG, the wavelength capacity C, and traffic demand matrix T, establish the minimum number of lightpaths to carry all traffic demands while allowing fractional lightpaths to exist between any pair of nodes.*

Denote l_{ij} (in the output lightpath set L) as the number of lightpaths to establish from node i to node j. The solution to *VTTR* could be obtained by rounding-up all real values of l_{ij} in *VTTR-rlx*. However, the round-up results in a large optimality gap. To strike a good balance between running time and solution quality, we develop a new algorithm to treat the integer constraints on lightpath variables l_{ij} as *lazy* constraints, and activate only a subset of them.

Consider the optimal solution $\{l_{ij}\}$ to the *VTTR-rlx* problem and the corresponding feasible solution $\{\lceil l_{ij} \rceil\}$ to *VTTR*, obtained by rounding up all the lightpath variables. Let us define: $U_{ij} = \frac{l_{ij}}{\lceil l_{ij} \rceil}, \;\; l_{ij} > 0$. The quantity U_{ij} represents the

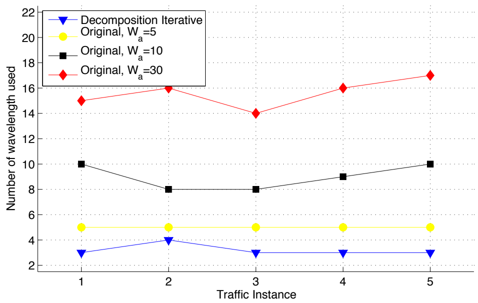

Fig. 1. Wavelength usage comparison

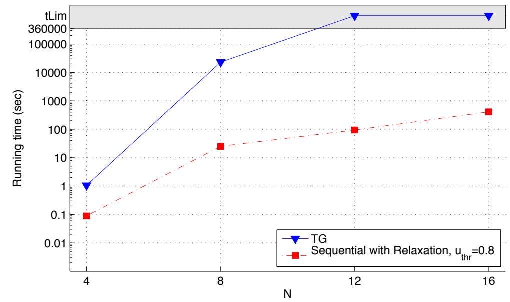

Fig. 2. CPU time comparison

utilization of the lightpaths from node i to node j in the rounded-up feasible solution. When the utilization is high (i.e., U_{ij} is close to 1.0), the corresponding lightpath resources are used effectively in the solution; furthermore, rounding up the corresponding lightpath variable to obtain a feasible solution makes only a small contribution to the optimality gap. Then, we develop our new relaxation algorithm as follows: 1). Solve *VTTR-rlx* to obtain the optimal solution and determine the corresponding feasible solution obtained by rounding up all non-integer lightpath variables. 2). Calculate U_{ij} for all non-integer lightpaths in the optimal solution and modify *VTTR-rlx* to activate the integrality constraints for the variables for which $U_{ij} \leq u_{thr}$, a predetermined threshold. 3). Determine the final solution by rounding up the modified *VTTR-rlx* outputs. We found out that by setting $u_{thr} = 0.8$, the solution in is within extra 5% of the optimal.

5. Numerical Results

Figure 1 plots the number of wavelengths used by solutions to the *TG* problem for five problem instances on a six-node ring network; each solution is obtained by providing the stated number W of wavelengths as input to the *TG* formulation. In addition, it also includes the number of wavelengths obtained by sequentially solving the *VTTR* and *RWA* subproblems on each of the five instances. As we can see, the sequential solution uses fewer wavelengths than any of the solutions to the original *TG* problem. Figure 2 compares the running time as a function of ring network size of new approach and basic formulations for solving the *TG* problem. As we can see, the new approach scales much better than the basic formulations, especially in larger networks.

6. Conclusion

We have presented a new decomposition approach to solve the traffic grooming problem sequentially, that scales well and enables network designers and operators to carry out extensive "what-if" analysis. The decomposition is exact when the network is not wavelength limited, and also minimizes the number of wavelengths used, avoiding the wavelength fragmentation issues of typical ILP formulations of the traffic grooming problem. We also developed an new partial LP relaxation that can be used to achieve a desired tradeoff between running time and solution quality.

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