Link Selection Algorithms for Link-Based ILPs and Applications to RWA in Mesh Networks

Zeyu Liu and George N. Rouskas

Outline

- Routing and Wavelength Assignment (RWA)
- Fast Link Formulation with Link Selection in Mesh
- Results and Conclusions

RWA: Problem Definition

- Input
  - Network topology
  - Traffic demands $T=[t_{ij}]$
- Output
  - The lightpaths (path and wavelength), one per connection request
- Constraints
  - Wavelength Continuity Constraint
    - assign the same wavelength on all links along the lightpath
  - Distinct Wavelength Constraint
    - assign lightpaths with common link(s) different wavelengths
- A tight coupling between RA and WA
  - NP-hard Problem

Solutions - Heuristics

- Decompose into subproblems
  - routing algorithm
    - Dijkstra's algorithm
    - edge disjoint shortest pair algorithm
  - wavelength assignment
    - first-fit, random-fit, etc
- Longest first alternate path (LFAP)
  - use alternate paths that cannot be established by shortest paths only
  - maximize the utilization of each wavelength
- ILP -> LP and rounding
- Disadvantage
  - hard to characterize the quality of solutions
  - may fail to find a feasible solution

Solutions – ILP Formulations

- Integer Linear Programming (ILP) Formulations -> can solve to optimality
  - link-based
    - Links as entities of interest
  - path-based
    - Pre-calculate the path candidates
  - maximal Independent Set (MIS)-based
    - Pre-calculate the path candidates and transfer into a multi-coloring problem

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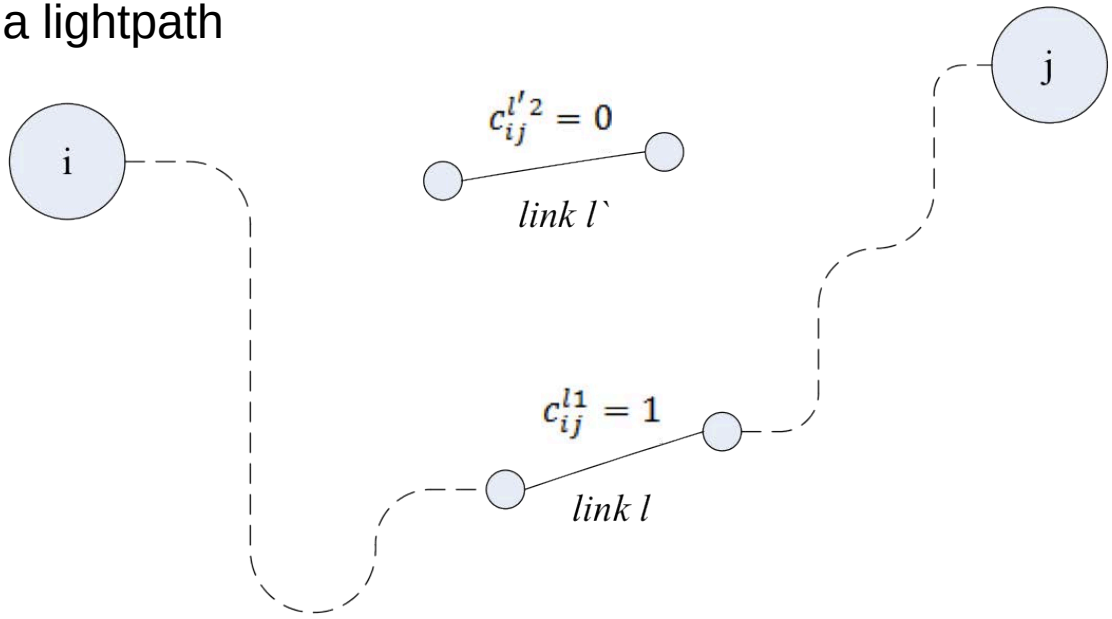
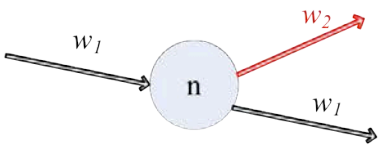
Link-based Formulation

- Links are the entities of interest
  - Natural advantage in solution quality
- Main set of decision variables
  - binary variables indicating whether a wavelength is assigned on a link for a lightpath

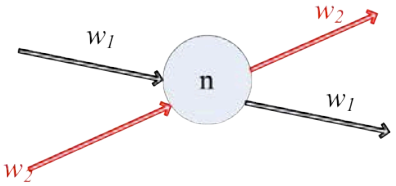


Link-based Formulation (2)

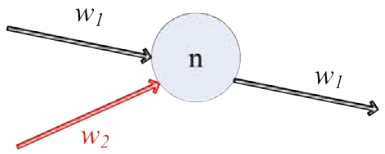
- Main set of constraints - multi-commodity flow equation



$n=i$



$n \neq i,j$



$n=j$

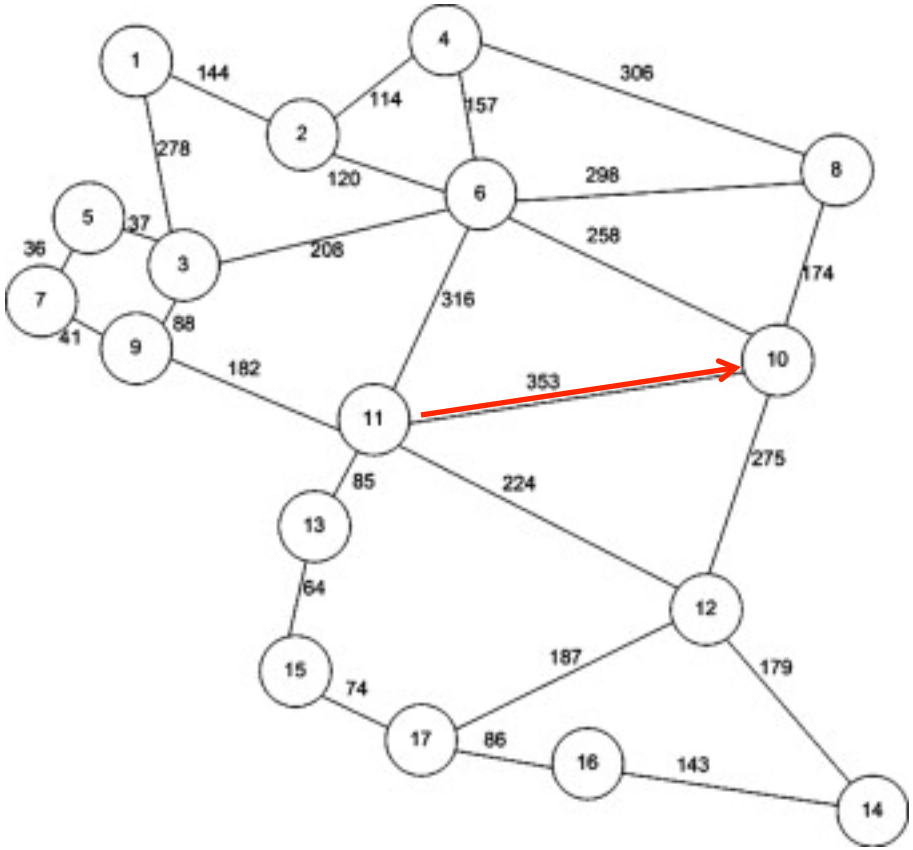
- Constraints
  - multi-commodity flow (implicitly ensure wavelength continuity constraint)
  - distinct wavelength constraints
  - traffic demands constraints

Link-based Formulation (3)

- Scalability Issue – large problem size
  - # of dominant variables
    - For each node pair, consider each link : $N^2|E|W$
  - # of dominant constraints
    - For each node pair, consider each node : $N^3\ t_{ij}$

Link-based Formulation (4)

- But, do we really need to consider each link?



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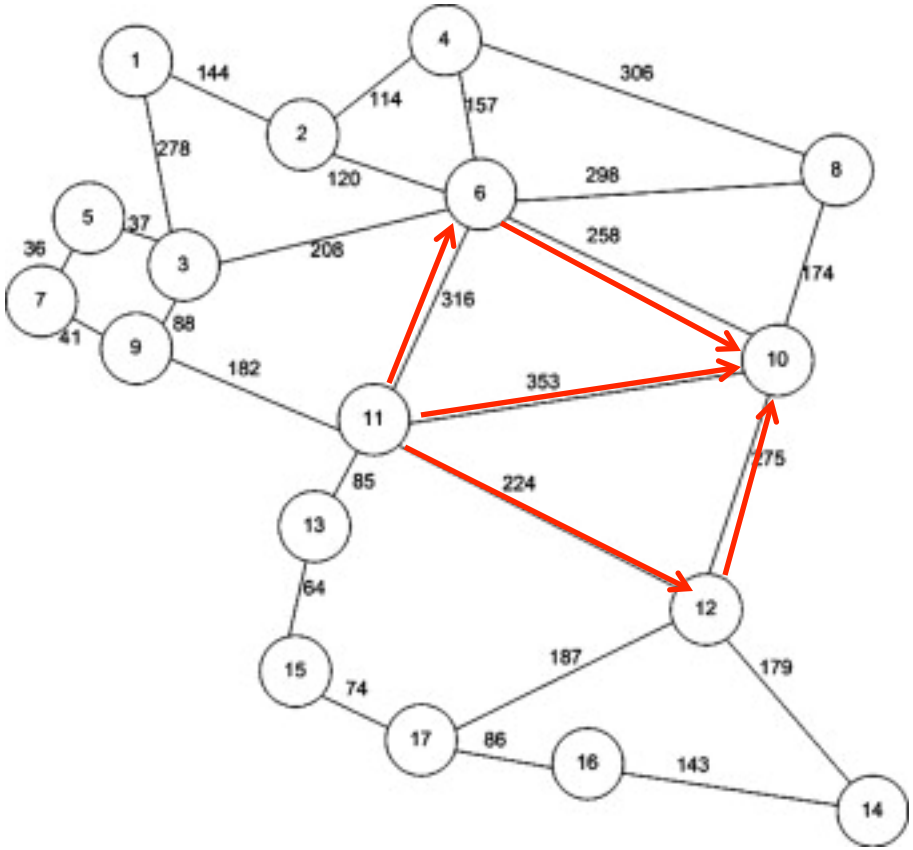

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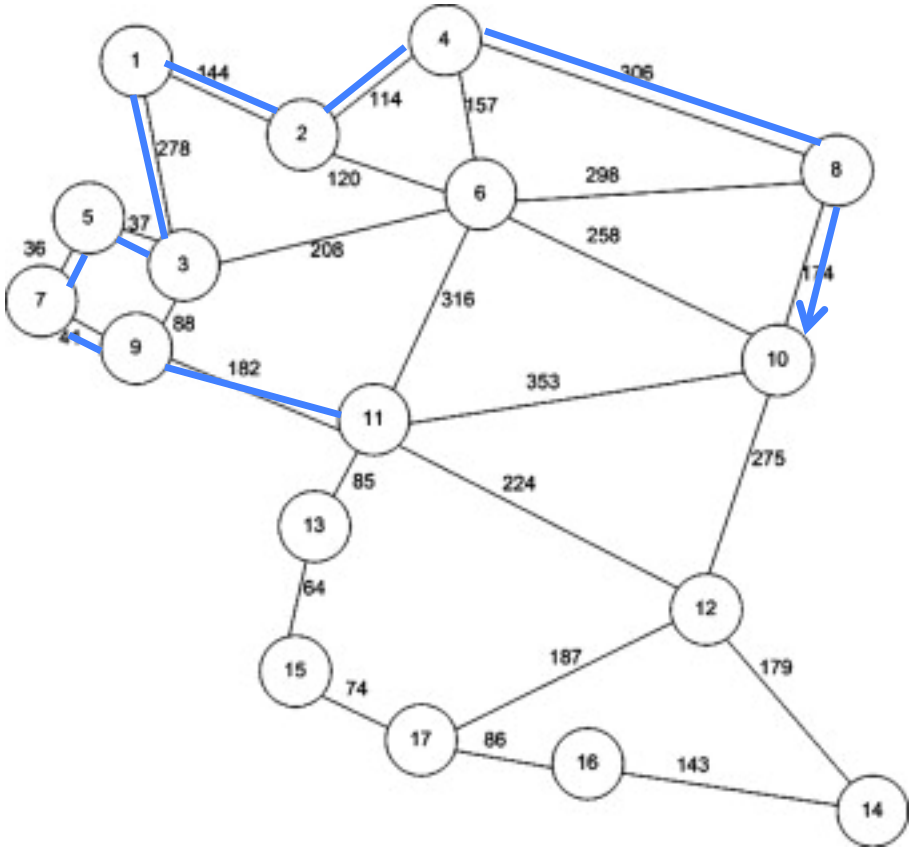


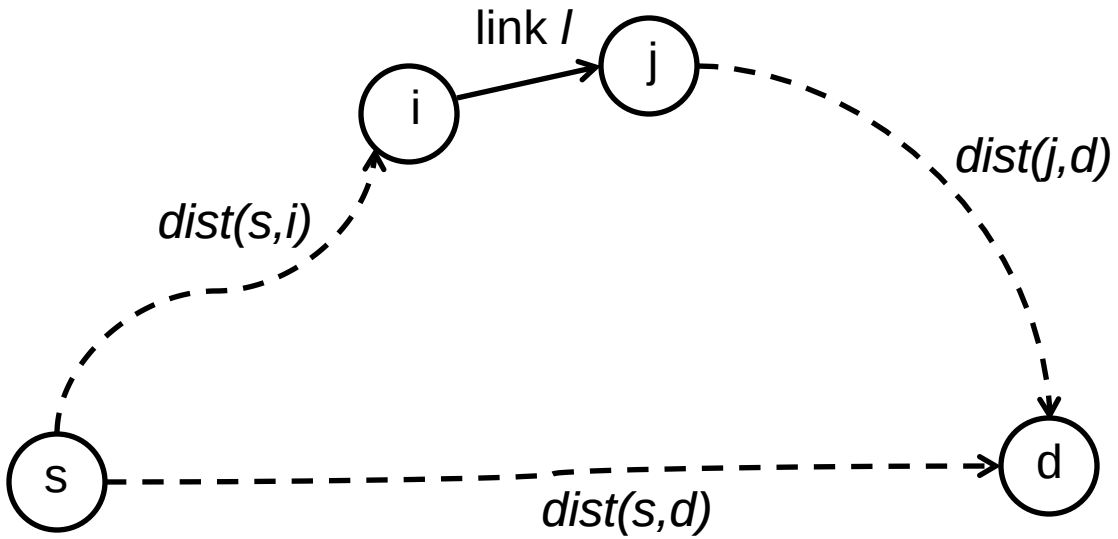
Link-based Formulation (4)

- But, do we really need to consider each link?
  - NO. Two reasons:
    - Waste network resources
    - Increase # of wavelengths needed.

Link Formulation with Link Selection

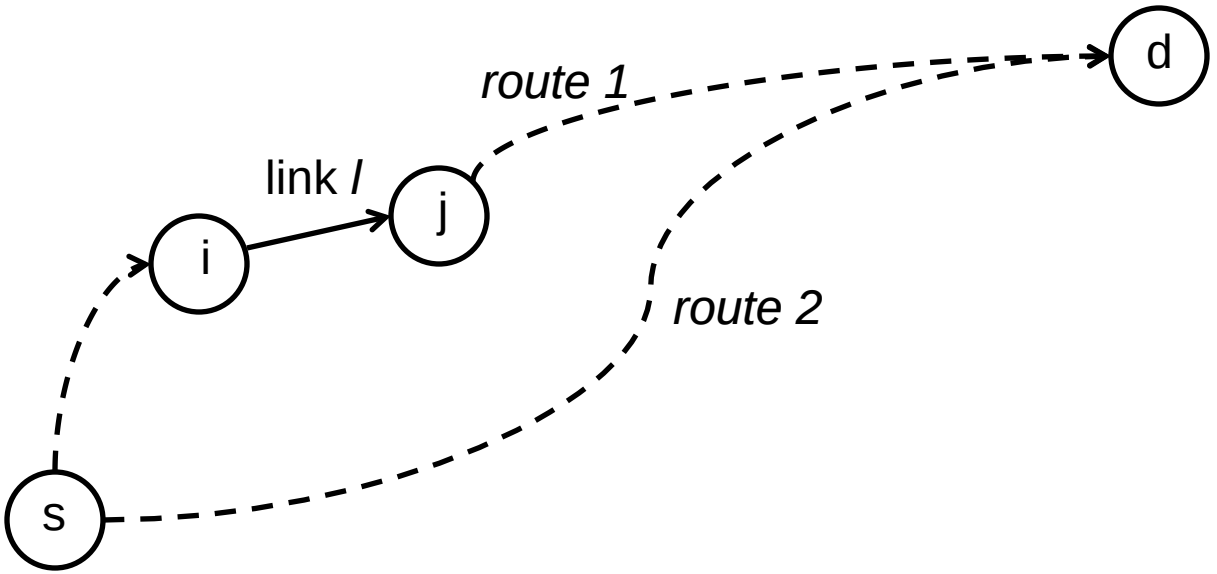
- Reduce the problem size – prune redundant variables
- Link selection algorithm 1 – *k-thres algorithm*

$$dist(s,i) + dist(j,d) + 1 \leq dist(s,d) + thres$$



Link Formulation with Link Selection (2)

- Reduce the problem size – prune redundant variables
- Link selection algorithm 2 – *k-path algorithm*
  - *Select links on the routes of k-shortest paths*



Link Formulation with Link Selection (2)

- Reduce the problem size – prune redundant variables
- Link selection algorithm 2 – *k-path algorithm*
  - More likely to give better solution
  - Tend to have less links selected as link reuse among the k paths

Link Formulation with Link Selection (3)

- Reduce the problem size – prune redundant variables
- Problem size decrease
  - # of variables
    - Only a fraction of all links is considered
  - # of constraints
    - Only nodes that are endpoints of the selected links

Link Formulation with Link Selection (4)

- Analysis 1 - # of variables vs. k
  - # of variables increases as k increases
  - *k-path* algorithm tends to have less # of variables

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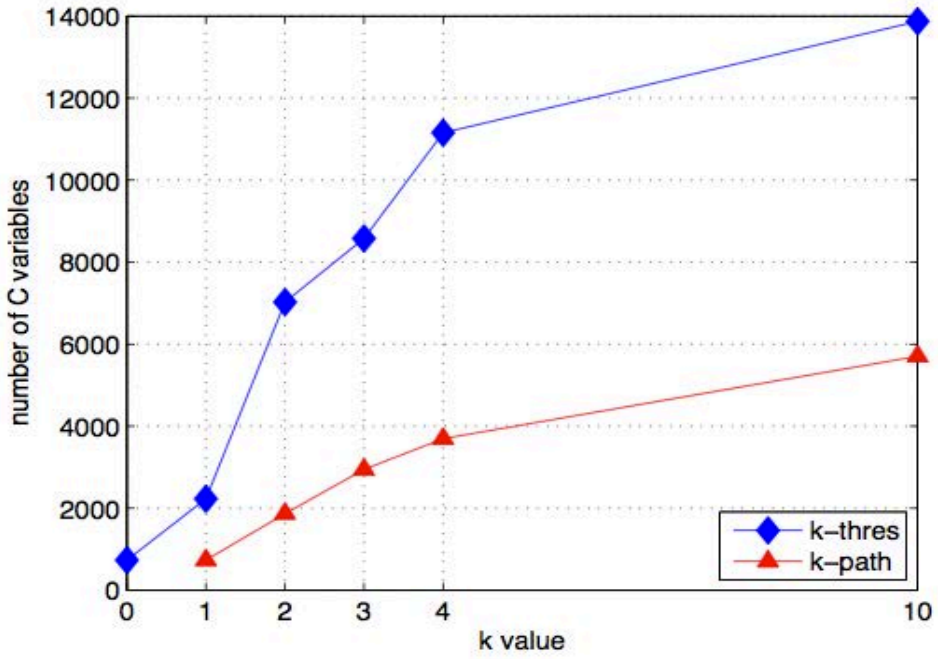

Figure 5.3: Number of c_{ij}^{wl} variables in German network using two link selection algorithms

Link Formulation with Link Selection (5)

- Analysis 2 - trade off between solution quality and running time
  - As the # of variables increases
    - Solution quality increases (# of wavelengths needed decrease)
    - Running time increases

Link Formulation with Link Selection (6)

- Analysis 3 - link formulation with link selection vs. path formulation

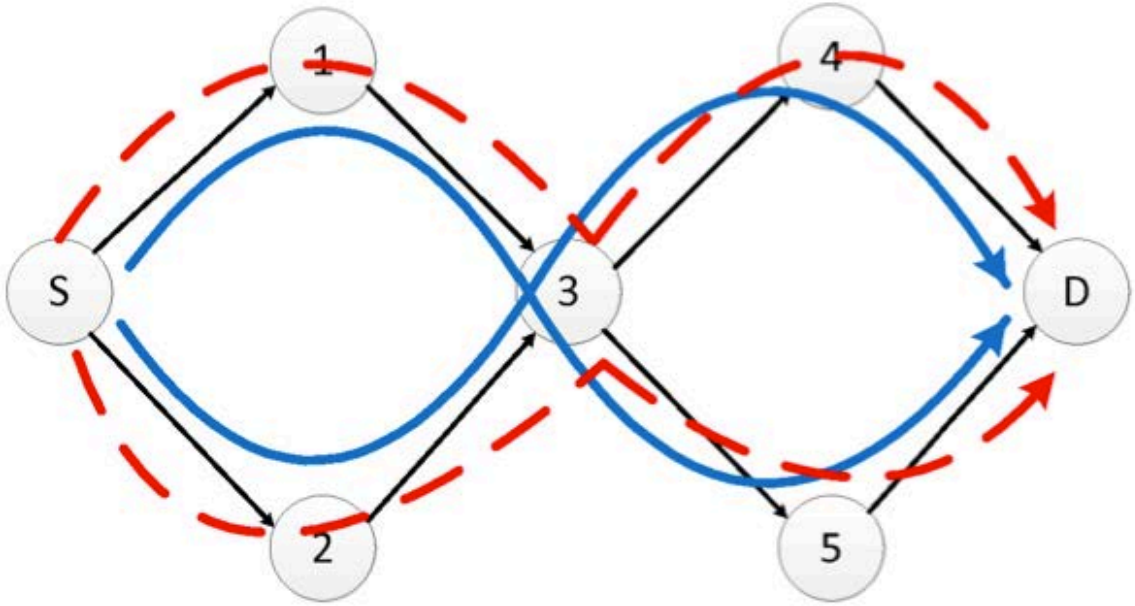

Figure 5.5: An illustration on the advantage of k-path link selection algorithm

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Link Formulation with Link Selection (8)

- Experimental study 1 – *k-thres vs. k-path*

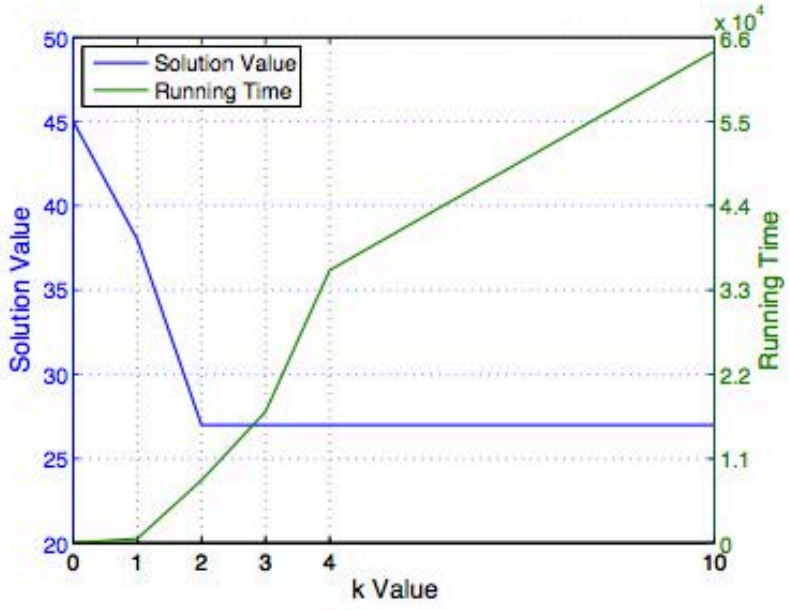

Figure 5.6: k-thres algorithm
Running time and solution value vs. k in German Network, $t_{max} = 2$

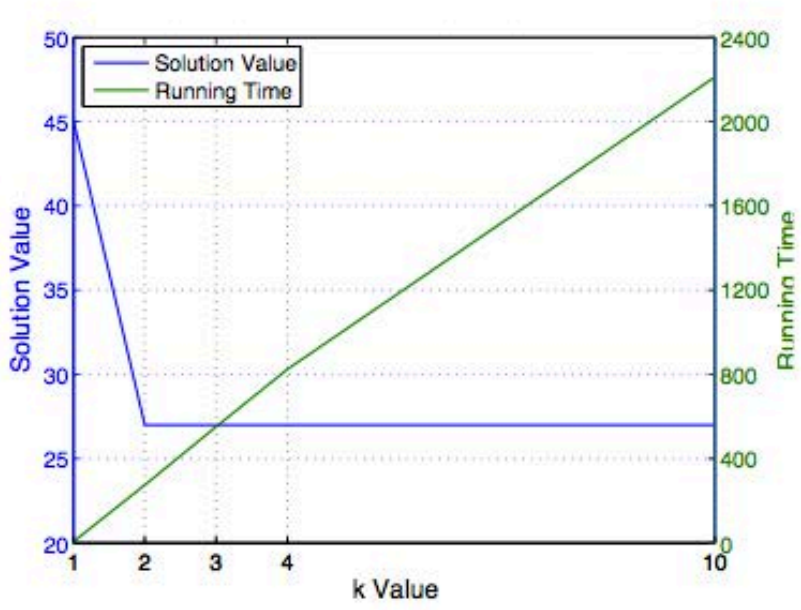

Figure 5.7: k-path algorithm

Link Formulation with Link Selection (8)

- Experimental study 1 – *k-thres vs. k-path*

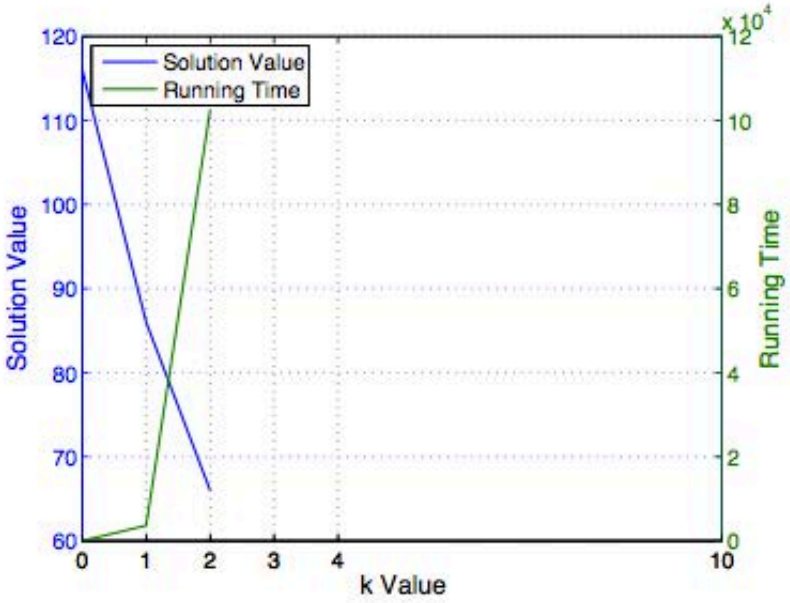

Figure 5.8: k-thres algorithm

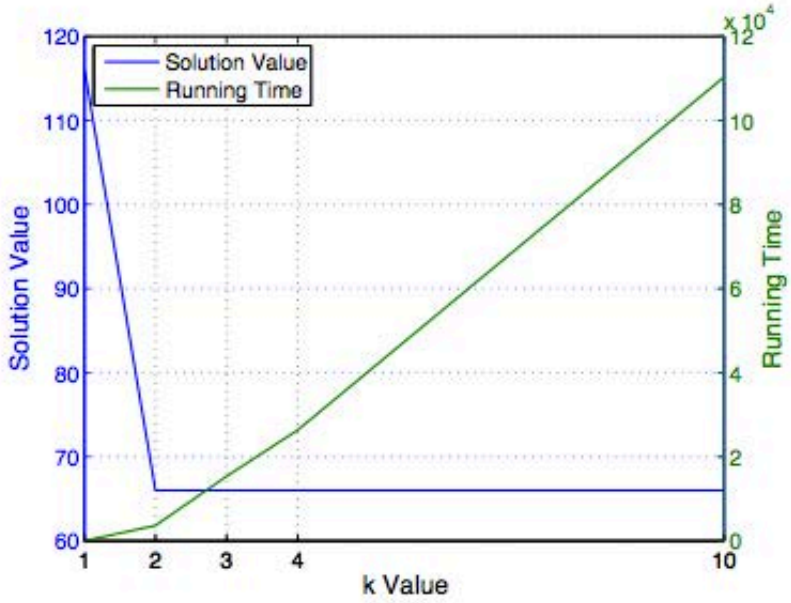

Figure 5.9: k-path algorithm

Running time and solution value vs. k in German Network, $t_{max} = 6$

Link Formulation with Link Selection (8)

- Experimental study 1 – *k-path in NSF network*

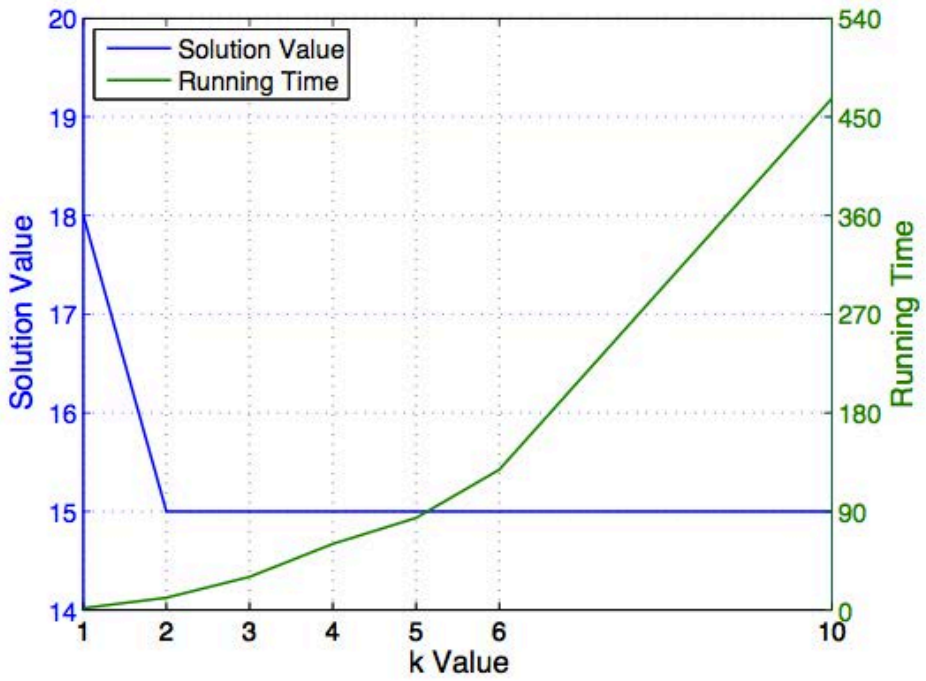

Figure 5.10: k-Path algorithm: Running time and solution value vs. k in NSF Network, $t_{max} = 2$

Link Formulation with Link Selection (8)

- Experimental study 1 – *k-path in NSF network*

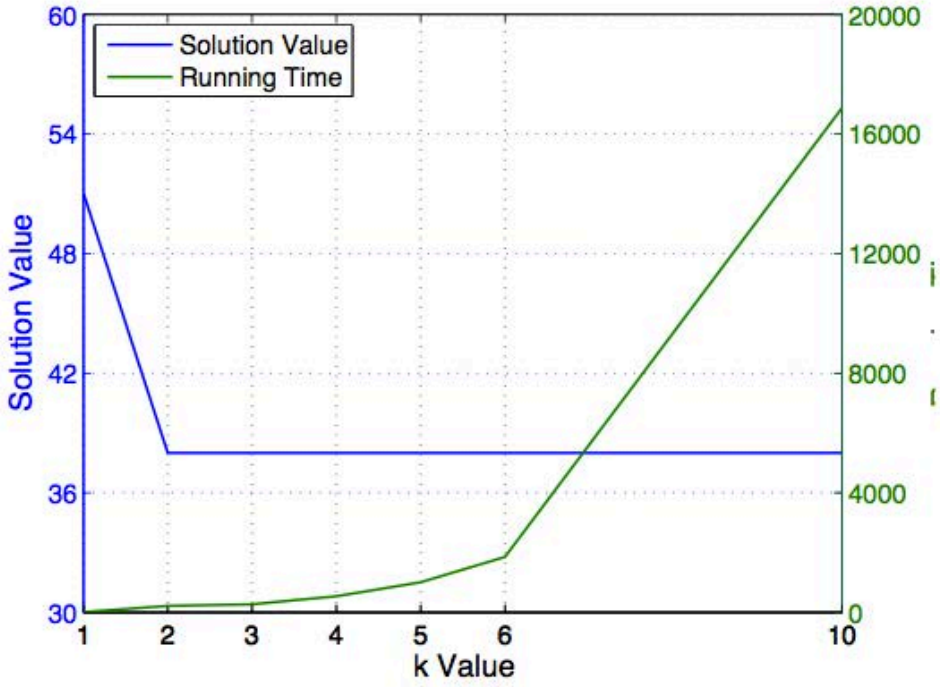

Figure 5.11: k-Path algorithm: Running time and solution value vs. k in NSF Network, $t_{max} = 6$

Link Formulation with Link Selection (9)

- Experimental study 2 – running time, compared with existing

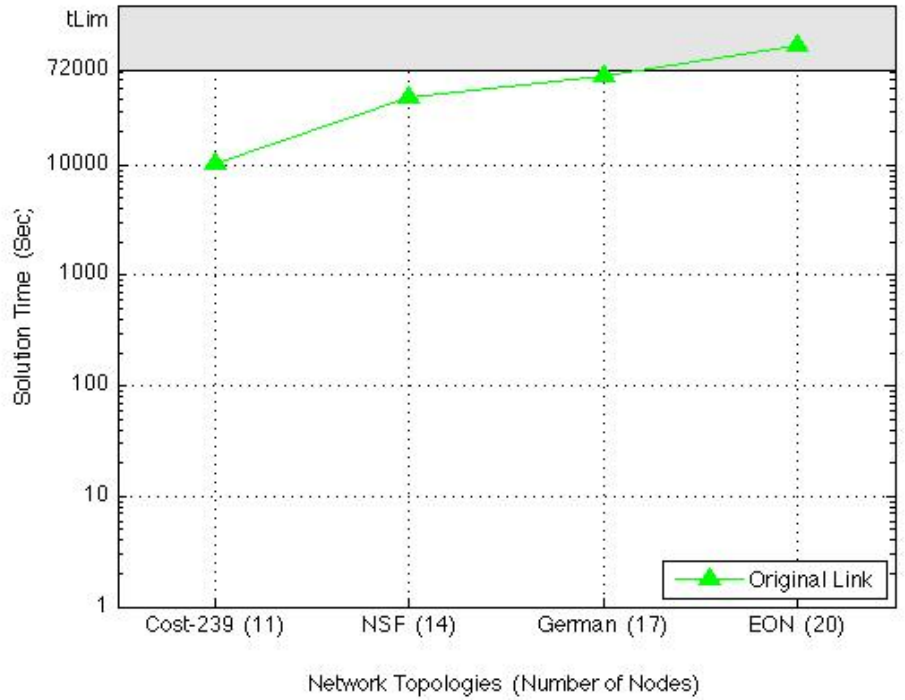

Figure 5.12: Running time of new link formulation with two link selection algorithms, compared with original link-based and path-based formulations

Link Formulation with Link Selection (9)

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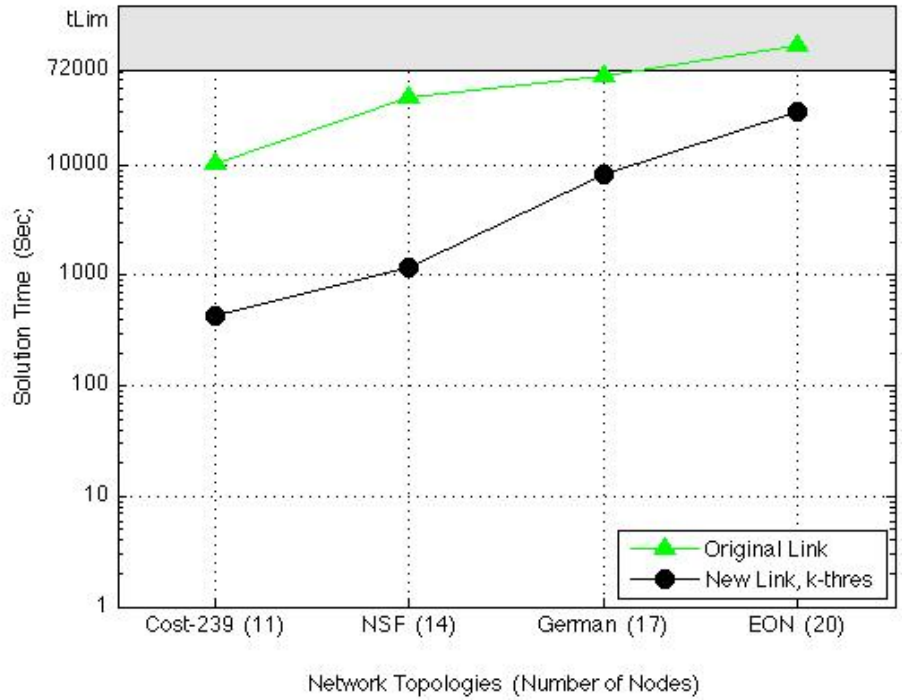

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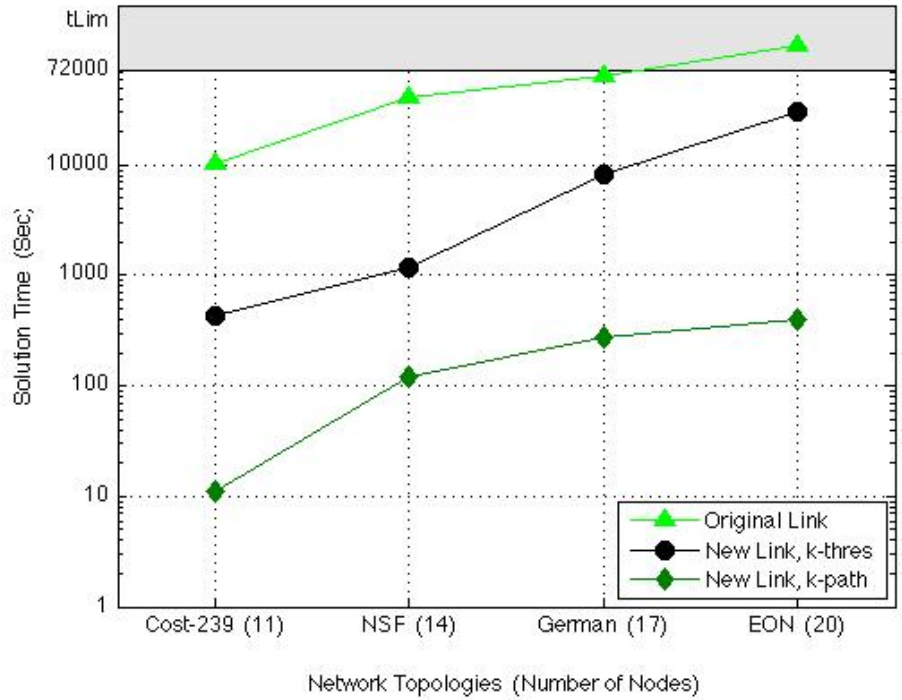

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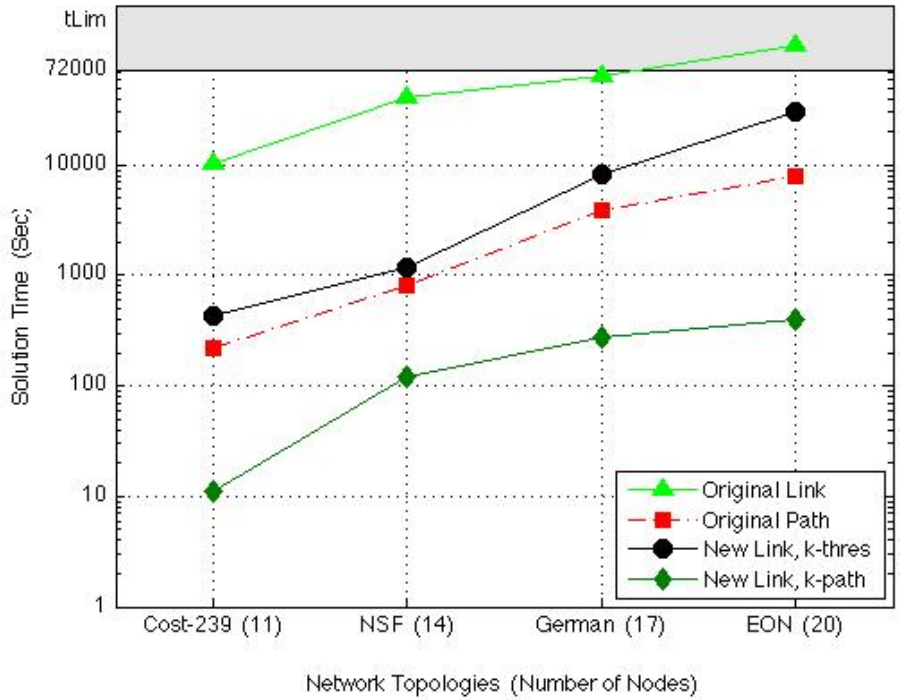

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Link Formulation with Link Selection (10)

- Experimental study 3 – solution quality, compared with existing

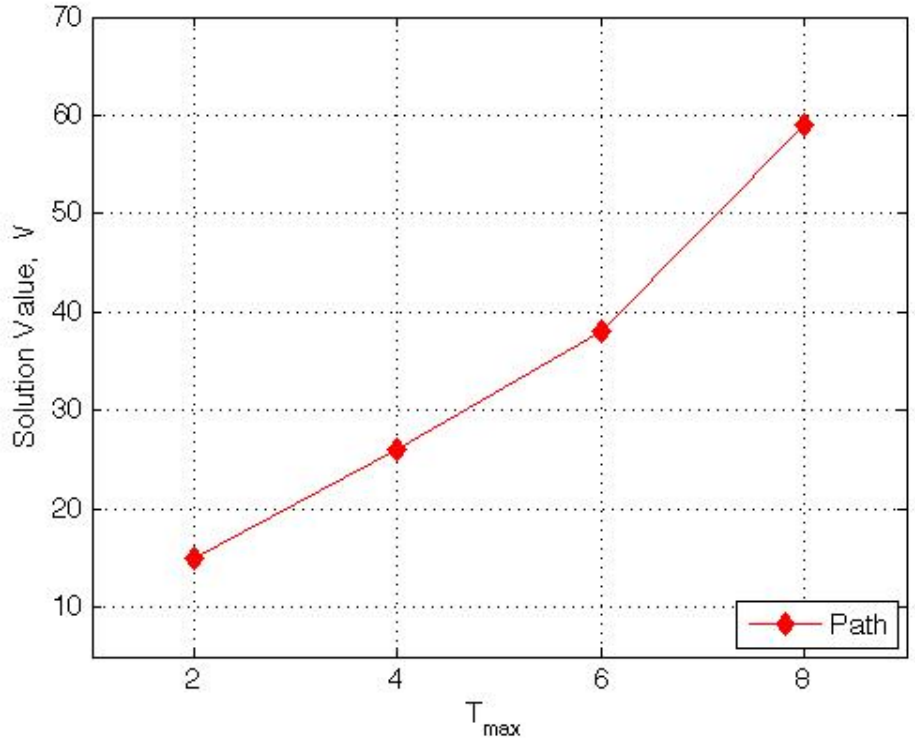
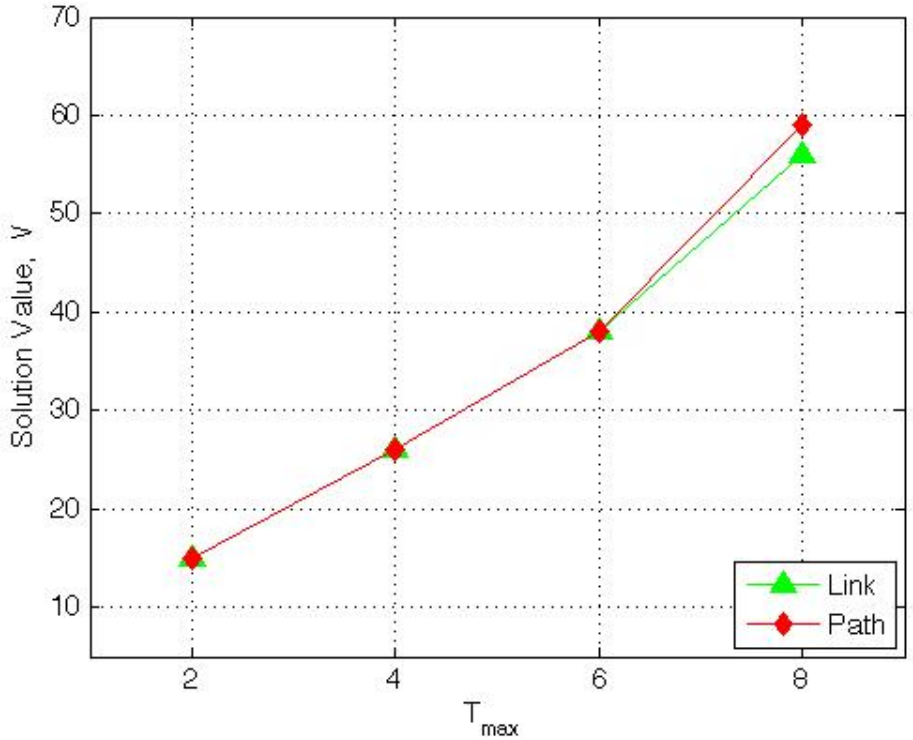


Figure 5.13: Solution value of new link formulation with two link selection algorithms against t_{max}, compared with original link-based and path-based formulations

Link Formulation with Link Selection (10)

- Experimental study 3 – solution quality, compared with existing



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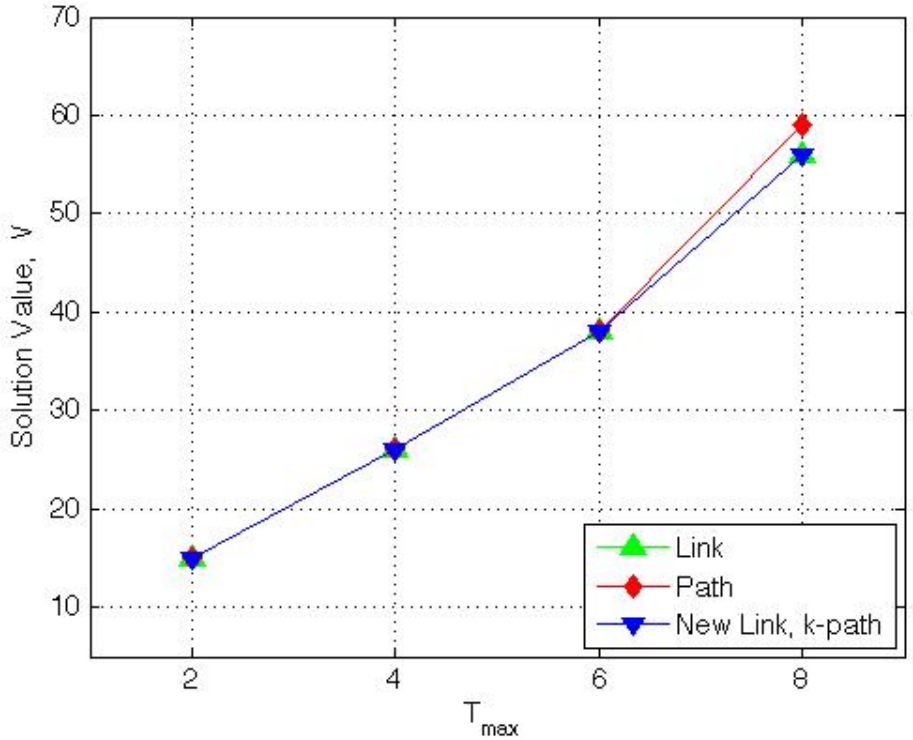

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Conclusions

- Benefits
  - greatly improved existing ILP formulation
  - possible to solve WDM networks representative to backbone and regional networks
  - speed up several optical network design problems that includes RWA as a subproblem (e.g., traffic grooming, survivability design etc.)
  - able to characterize the performance of heuristics and develop new efficient ones

Future Work

- Speeding up traffic grooming in mesh
  - One direction is to extend link selection to grooming assignment
- Apply link selection to:
  - other flow-based problems
  - problems with path constraints (e.g., impairment-aware RWA)